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**MOMENT-BASED  
FATIGUE LOAD MODELS  
FOR WIND ENERGY SYSTEMS**

**LeRoy M. Fitzwater, Steven R. Winterstein**  
*Civil & Envir. Eng. Dept., Stanford University*

**Lance Manuel**  
*Civil Eng. Dept., University of Texas at Austin*

**Paul S. Veers**  
*Wind Energy Tech. Dept., Sandia National  
Laboratories*

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# Stochastic Fatigue Analysis

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## Fatigue: What We Care About:

- $R$  = stress ranges from random time history  
(e.g., rainflow-counted from measured or simulated history)
- Mean Fatigue Damage:

$$E[D] \propto E[R^b] \quad \text{where}$$

$E[\cdot]$  = average (“expected” value)

$b$  = material parameter from  $S$ - $N$  curve; e.g.,

*Welded steels:*  $b \approx 3$

*Fiberglass composites:*  $b \approx 9$

## Resulting Complications:

- Choice of material affects how “tail-sensitive” to rare loads; hence optimal modelling given limited data
- Complicated dynamics of wind turbines: rotational sampling of wind field, gravity plus turbulence effects, bimodal probability densities of  $R$

# Moment-Based Fatigue Load Models

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- **Standard Weibull model** of  $R$  (two moments)
  - *Definition:*  $P[R > r] = \exp[-(\frac{r}{\alpha_R})^{\beta_R}]$
  - *Parameters:*  $\alpha_R, \beta_R$
  - *Preserves:*  $E[R]$  and  $E[R^2]$  (or  $\sigma_R^2$ )
- **Quadratic Weibull model** of  $R$  (three moments)
  - *Starts with:*  $W$  = standard Weibull model of  $R$
  - *Definition:*  $R = R_0 + \kappa[W + \epsilon W^2]$
  - *Parameters:*  $R_0, \kappa, \epsilon$
  - *Preserves:*  $E[R], E[R^2],$  and  $E[R^3]$
- **Damage-based Weibull model** of  $R$ 
  - *Definition:* Apply Weibull model to  $W = R^{b_{min}}$  where  $b_{min}$  = minimum value of  $b$  to be expected (e.g.,  $b_{min} = 3-5$ ).
  - *Preserves:*  $E[R^{b_{min}}]$  and  $E[R^{2b_{min}}]$

# TRADEOFFS: Moment-Based Models

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- **Standard Weibull model** of  $R$  (two moments)
  - Standard relations to fit parameters to moments
  - Preserves  $E[R]$  and  $E[R^2]$  → requires least data
  - May not represent upper tail accurately
- **Quadratic Weibull model** of  $R$  (three moments)
  - Preserves  $E[R]$ ,  $E[R^2]$ , and  $E[R^3]$  → can capture non-Weibull trends in upper tail
  - Requires more data than ordinary Weibull
  - Needs numerical algorithm to estimate parameters from moments. *Algorithms:* FITS, MAXFITS
- **Damage-based Weibull model** of  $R$ 
  - Preserves  $E[R^{b_{min}}]$ ,  $E[R^{2b_{min}}]$  (e.g.,  $b_{min}=3-5$ )
  - Weibull fit to  $R^{b_{min}}$ ; standard relations between parameters and moments
  - Preserves damage exactly if  $b=b_{min}$  or  $b = 2b_{min}$  (presumably good accuracy for intermediate  $b$  values)
  - Most tail sensitive; e.g., matches moments 3 and 6, or 4 and 8, ..... → requires most data

# Description of Field Data

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## Data from Instrumented Wind Turbine:

- 110kW Horizontal Axis Wind Turbine (WINCON 110XT) in Lavrio, Greece
- Wind characterized by
  - $\bar{V}$ =mean wind speed
  - $I=\sigma_V/\bar{V}$ =turbulence intensity
- 101 10-minute samples ( $15 \leq \bar{V} \leq 19$  m/s)
- In-plane (edge) and Out-of-plane (flap) bending measured
- Histograms of  $R$ =rainflow-counted bending moment ranges provided

# Long-Term Analysis

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- Characterize each necessary range moment  $\mu_i$  by power-law regression:

$$\mu_i = a_i \left( \frac{V}{V_{ref}} \right)^{b_i} \left( \frac{I}{I_{ref}} \right)^{c_i}$$

(e.g.,  $i=1,2,3$  for quadratic Weibull)

- Estimate uncertainty in regression parameters  $\theta$  given limited data:

$$\theta = [a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3]$$

- Resulting long-term distribution of fatigue ranges  $R$ :

$$F(r|\theta) = \int_{all \bar{V}} \int_{all I} F(r|\bar{V}, I|\theta) d\bar{V} dI$$

- Simulation of many  $\theta \rightarrow$  range of possible  $F(r|\theta)$ ; rank to provide confidence intervals of  $F$  for fixed  $r$ .

# Summary and Conclusions

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- **Set of Models** (increasingly tail-sensitive):

Model	Quantities Preserved:
Standard Weibull	$E[R], E[R^2]$
Quad Weibull	$E[R], E[R^2], E[R^3]$
Damage-based Weibull	$E[R^{b_{min}}], E[R^{2b_{min}}]$
Quad Damage-based Weib	$E[R^{b_{min}}], E[R^{2b_{min}}], E[R^{3b_{min}}]$

- **Most Efficient:** Quadratic Weibull
  - Least tail-sensitive generalization of Weibull
  - Parameters found numerically; often need data threshold
- **Simplest to Automate:** Damage-based Weibull
  - No subjective choice of threshold, etc.; ensured to give consistent damage ( $b_{min} \leq b \leq 2b_{min}$ )
  - Simple parameter estimation (Weibull)
  - Can be quite tail-sensitive