Moment Based Air Gap Models For Floating Structures

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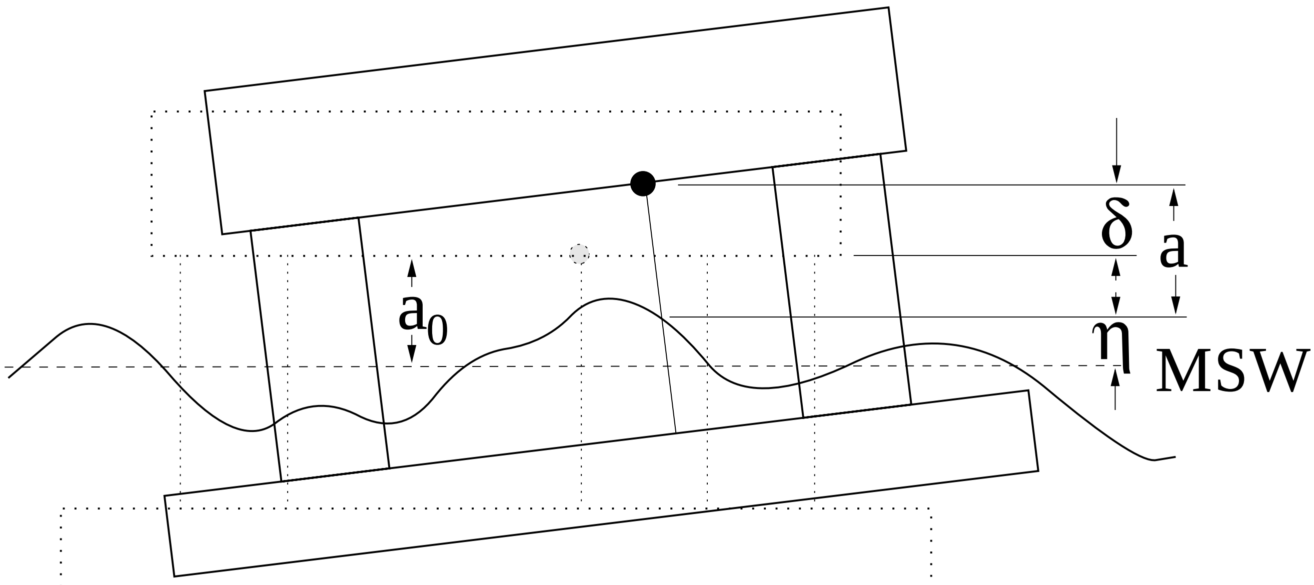
Air Gap Notation



For each wave probe location (x, y):

$\eta(t) \equiv$ Water Surface Elevation

$\delta(t) \equiv$ Net Vertical Displacement

$\mathrm{a}_0 \equiv$ Still-water Air Gap

$\mathrm{a}(t) \equiv$ Air Gap Response

Air Gap - A Basic Understanding

Air Gap Response:

$$a(t) = a_0 - [\eta(t) - \delta(t)] = a_0 - r(t)$$

$$r(t) = \eta(t) - \delta(t)$$

$r(t) > a_0$ (or $a < 0$) indicates deck impact

Motions (The Easy Part):

$$\delta(x, y, t) = \xi_3(t) - x \cdot \sin(\xi_4(t)) + y \cdot \sin(\xi_4(t))$$

Waves (The Hard Part):

$$\eta = \eta_1 + \eta_2$$

$\eta_1 = \eta_1^I + \eta_1^D$ (first order)

$\eta_2 = \eta_2^I + \eta_2^D$ (second order)

Calculation of η: Theory

First-Order Part:

$\eta_1 = \eta_1^I + \eta_1^D$

$\eta_1^I \equiv$ incident first-order wave

η_1^D from first-order diffraction

(e.g. first-order `WAMIT`)

Second Order Part:

$\eta_2 = \eta_2^I + \eta_2^D$

$\eta_2^I \equiv$ incident second-order wave

(second-order Stokes theory)

η_2^D from second-order diffraction

(e.g. second-order `WAMIT` ($$$))

Calculation of η: Hope

Perhaps η_2^D is not necessary:

Calculate air gap statistics directly from:

First-order diffraction theory

(structure-dependent)

Stokes second-order theory

(structure-independent)

Two Approaches:

Moment-based model

Ignore second-order diffraction effects on α_3 and α_4.

Narrow-band model

Ignore second-order diffraction effects on α_3 and α_4 on maximum crest only.

Associated Work

OMAE 1999

Steve Winterstein and Bert Sweetman:

Air Gap Response of Floating Structures:
Statistical Predictions vs Observed Behavior
Paper No. OMAE 99-6042

η_1^{D} **Only; no** η_2^{I} **or** η_2^{D}

Lance Manuel and Steven Winterstein

Air Gap Response of Floating Structures under Random Waves: Analytical Predictions based on Linear and Nonlinear Diffraction
OMAE 99, Oral Presentation Only

Complete η_2^{I} **and** η_2^{D}

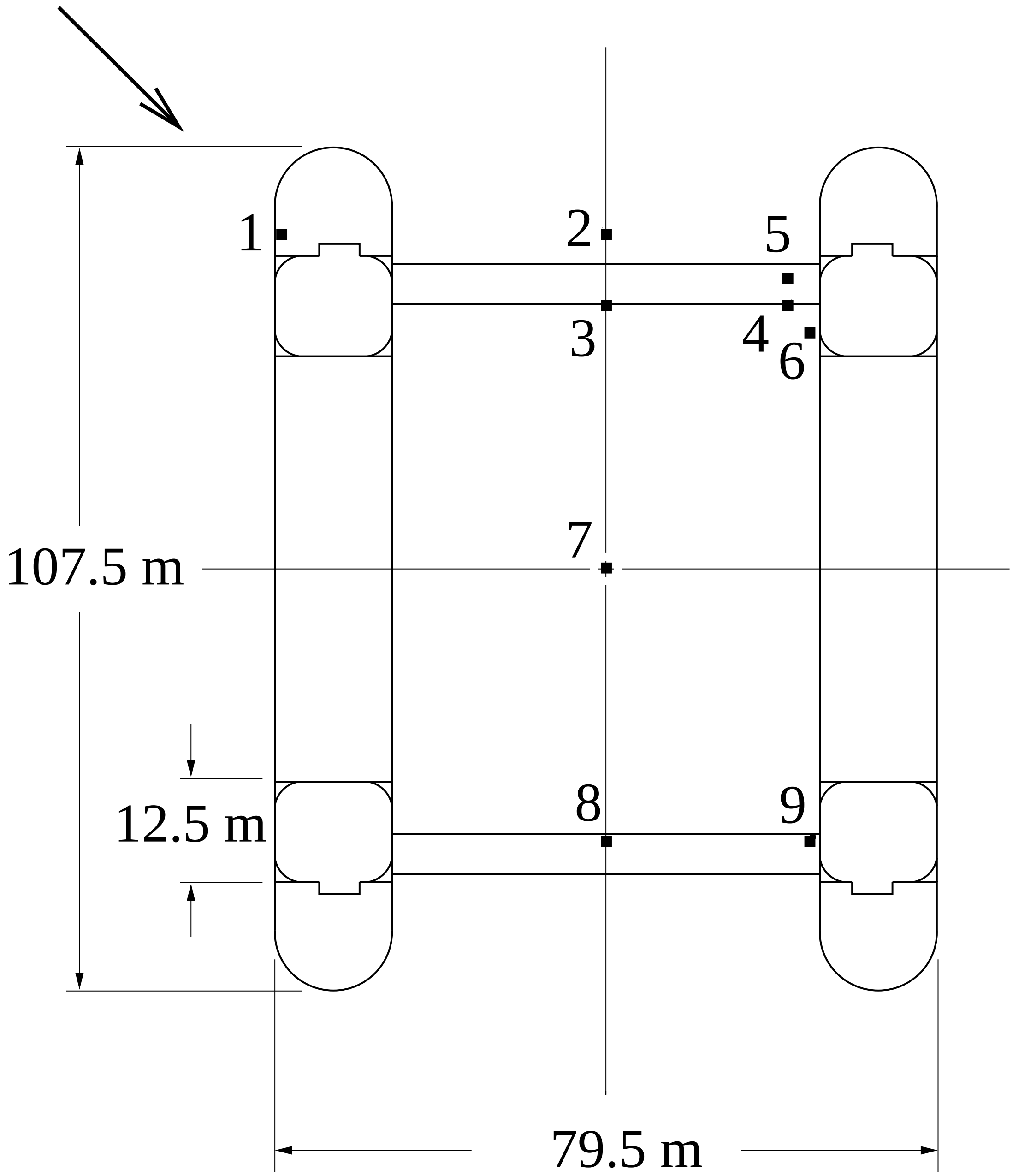
Here: ETCE/OMAE 2000

Bert Sweetman and Steve Winterstein:

Moment Based Air Gap Models For Floating Structures

η_2^{I} **and** η_1^{D}**; no** η_2^{D}

Veslefrikk Platform



Incident Wave Direction and
Numbered Locations of Air Gap Probes

Veslefrikk Semi-Submersible

Platform Particulars	
Length Over All (LOA):	107.50 m
Longitudinal Column Spacing:	68.00 m
Transverse Column Spacing:	67.00 m
Column Length w/o Sponson:	12.50 m
Column Breadth:	12.50 m
Pontoon Breadth:	14.25 m
Pontoon Height:	9.50 m

Survival Conditions	
Draft, D:	23.00 m
Displacement:	40,692 tonnes
Airgap to Still Water Level:	17.50 m
Center of Gravity (from keel):	24.13 m
Pitch Radius of Gyration:	33.76 m
Roll Radius of Gyration:	34.26 m
Transverse Metacentric Height:	2.36 m
Water Depth:	175.00 m

Model Test Sea-states

H_S [m]	T_P [s]	γ	Number of 3-hour tests	Spectral Type
12.0	11.5	4.0	5	Bimodal
14.0	13.5	3.0	6	JONSWAP
15.5	16.5	2.0	2	JONSWAP

Calculation Method(TFPoP)

Input

Deterministic Transfer Functions

Wave Power Spectral Density (e.g. JONSWAP)

Output:

Extreme Statistics

Solution Strategy:

1) Find First Four Moments (m, σ, α_3, α_4)

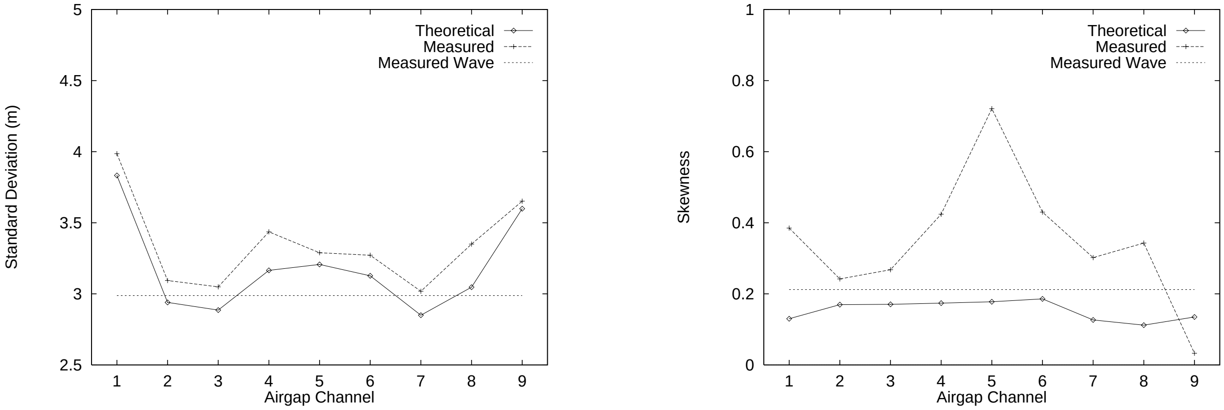
2) Use Hermite Transformation to Estimate Extreme Statistics from Moments

Hope:

1) Physical Theory Adequate to Estimate α_3 and α_4

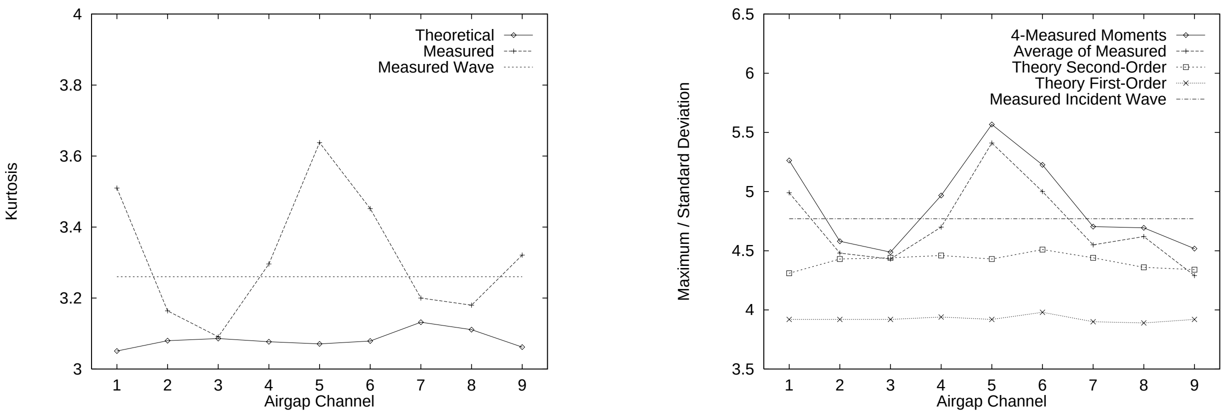
2) Statistical Theory Adequate to Estimate Extremes from α_3 and α_4

Moment-Based Calculation Results



Standard Deviation

Skewness



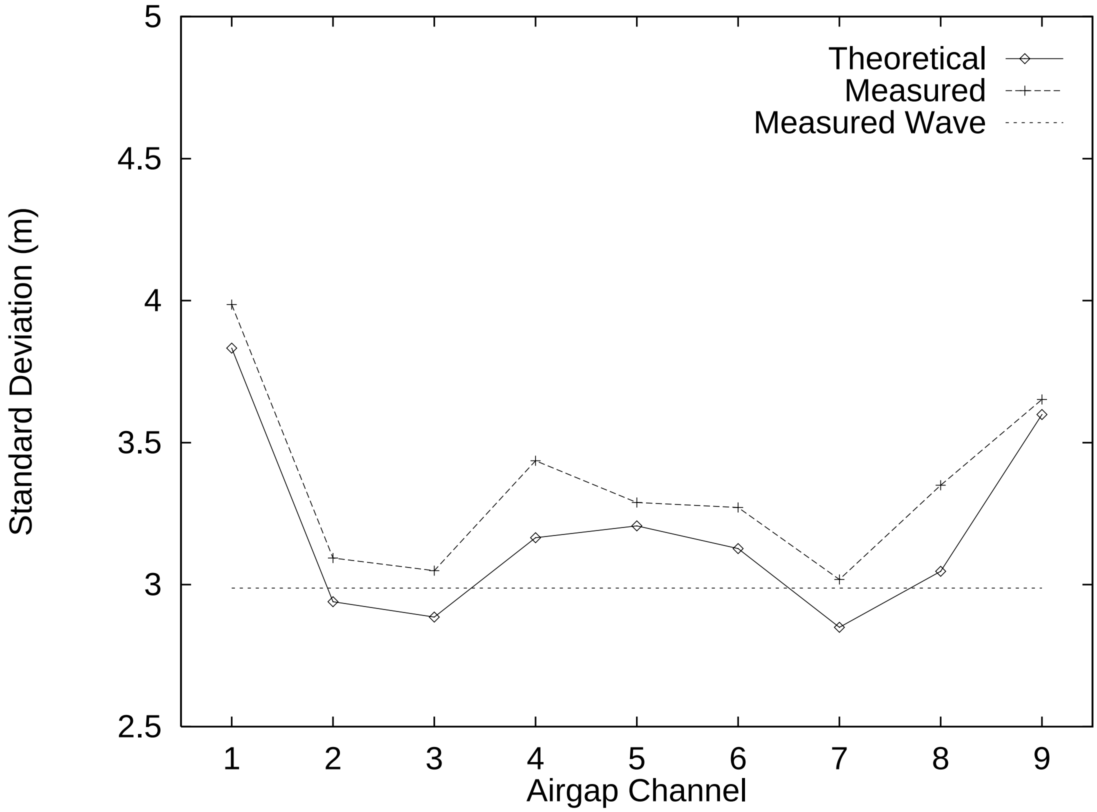
Kurtosis

Peak Factor

Wave and Airgap Response Processes

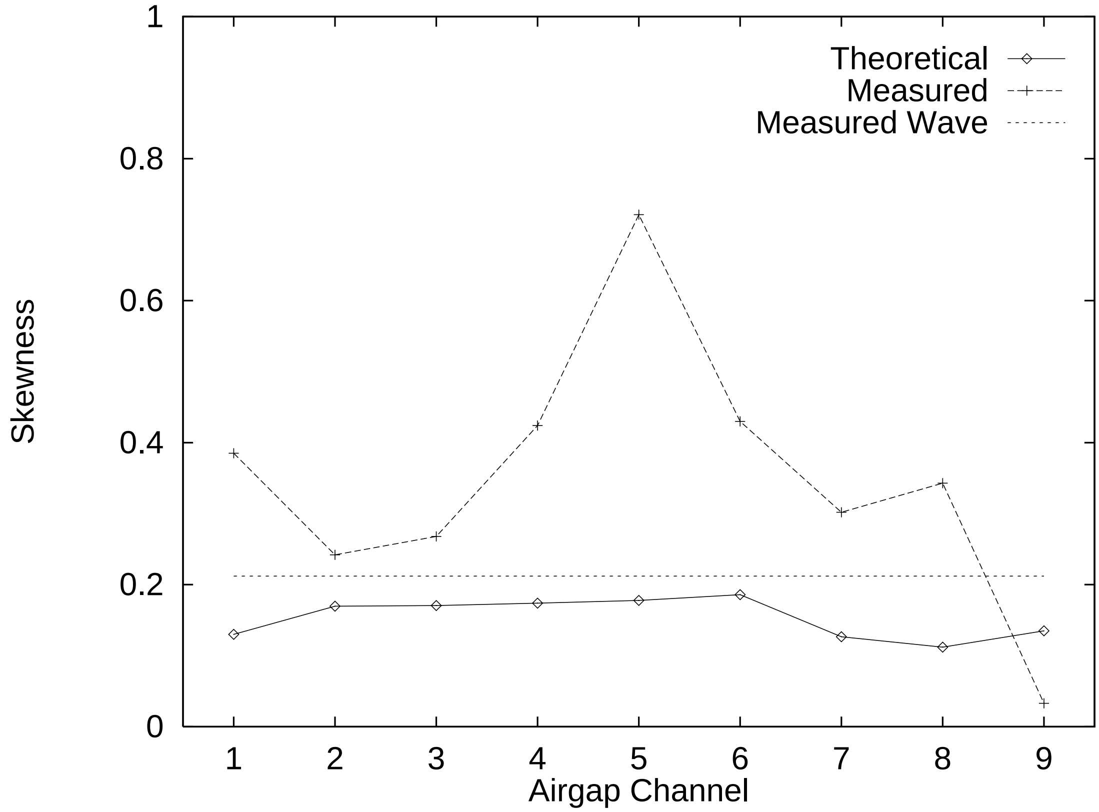
H_s=12 meters, T_p=11.5 seconds

Moment-Based Calculation Results



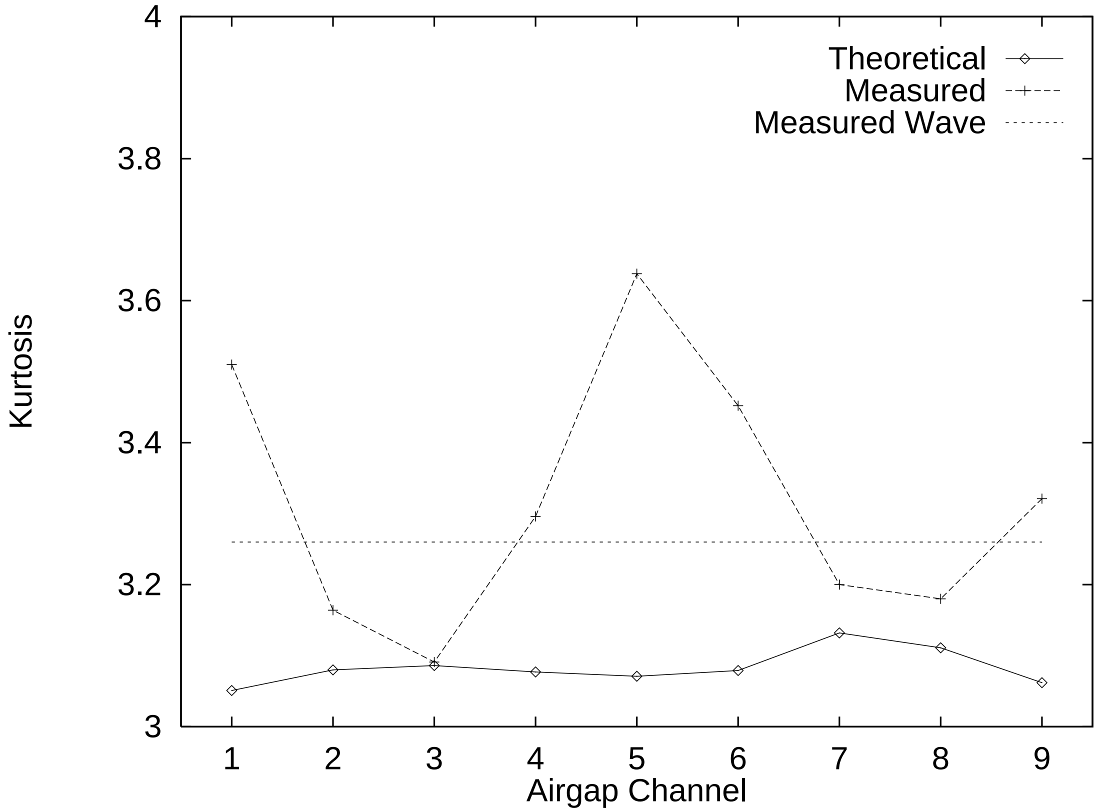
Standard Deviation

Moment-Based Calculation Results



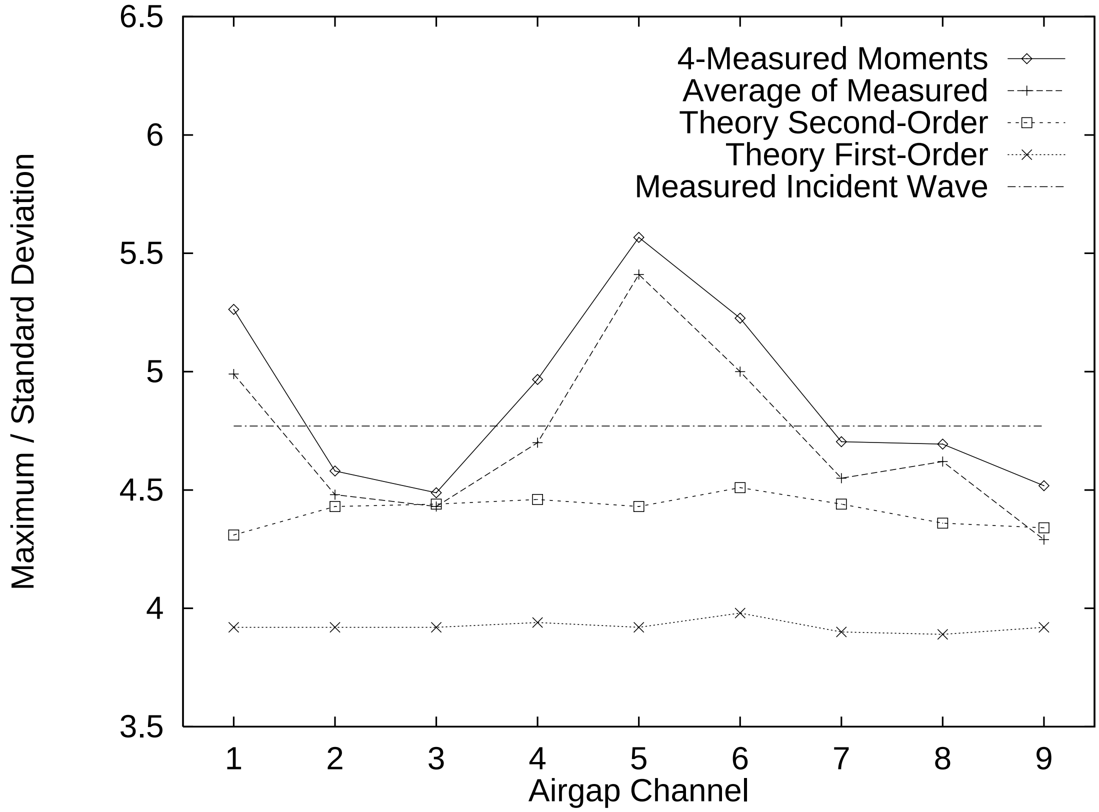
Skewness

Moment-Based Calculation Results



Kurtosis

Moment-Based Calculation Results



Peak Factors

Narrow-Band Approximation

First-order process (Gaussian vibration theory):

$$\eta_1(t) = a(t)\cos[\omega t + \theta(t)]$$

Second-order contribution (assumed phase-locked):

$$\eta_2(t) = a^2(t) H_2^+(\omega,\omega)\cos 2[\omega t + \theta(t)]$$

From Stokes Wave Theory:

$$H_2^+ = \frac{k}{2}$$

Combined Result when $\eta_1 = \eta_{1,\max}$:

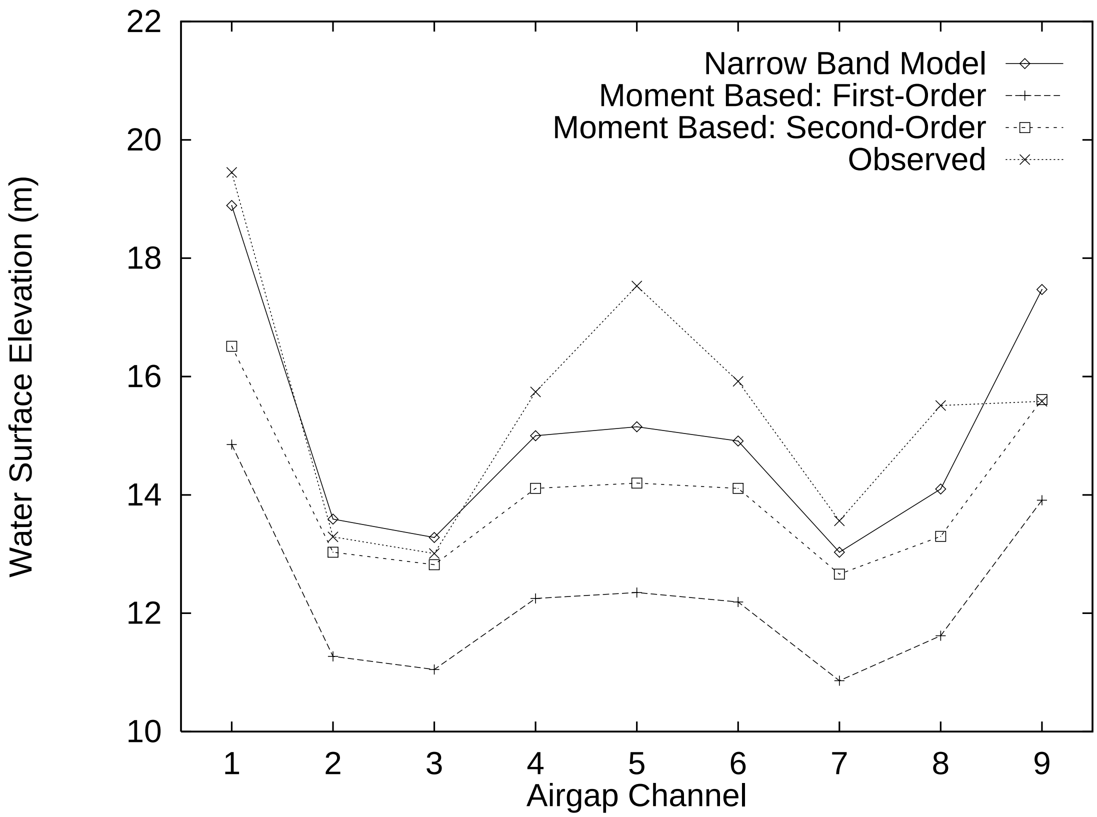
$$\begin{aligned} E[\eta_{max}] &= E[\eta_{1,max}] + E[\eta_{1,max}^2] H_2^+(\omega,\omega) \\ &\approx E[\eta_{1,max}] + E[\eta_{1,max}^2]\frac{k}{2} \end{aligned}$$

where:

$$\begin{aligned} E[\eta_{1,max}] &= \sigma_{\eta_1}[\sqrt{2\ln N} + \frac{0.577}{\sqrt{2\ln N}}] \\ E[\eta_{1,max}^2] &= E[\eta_{1,max}]^2 + \sigma_{\eta,max}^2 \end{aligned}$$

Narrow-Band Results:

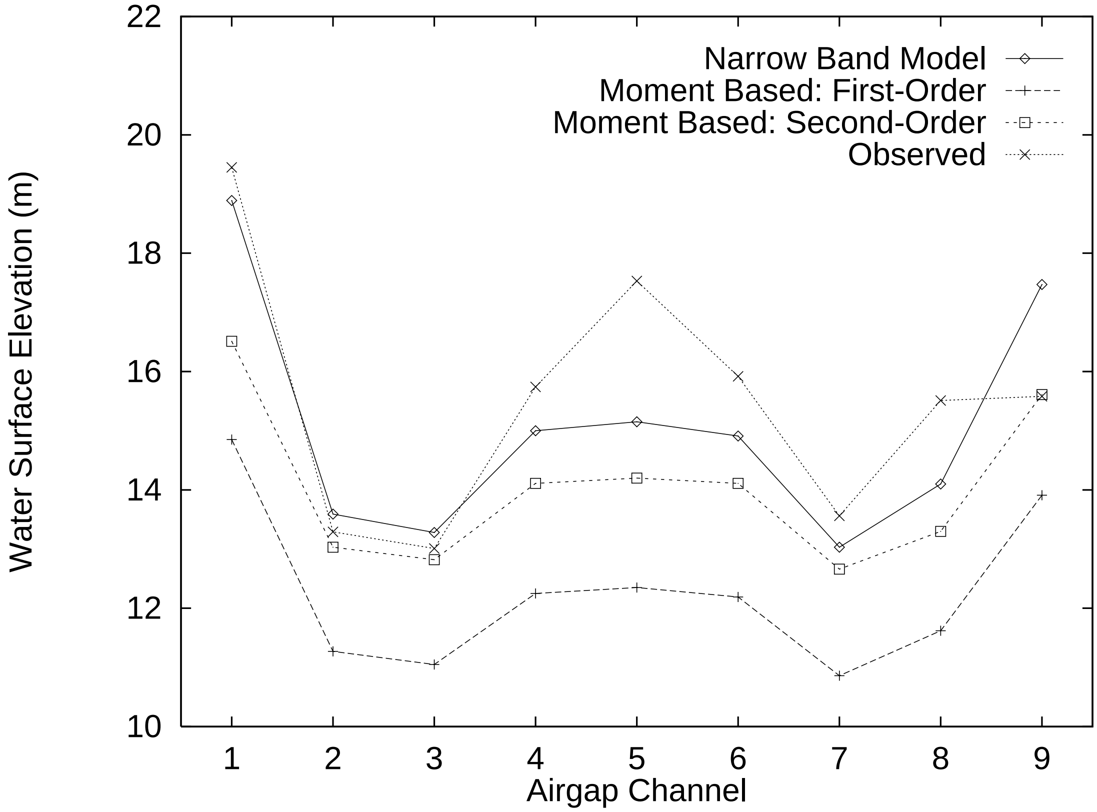
H_s = 12 m, T_p = 11.5 seconds, Bimodal



η_{max}: Observed versus Predicted

Narrow-Band Results:

$H_s = 14$ m, $T_p = 13.5$ seconds, JONSWAP



η_{max}: Observed versus Predicted

Conclusions

A standard linear diffraction model is found to noticeably under-predict extreme airgap demand ($\sim 20\%+$), while only moderately under-predicting RMS demand ($\sim 10\%-$).

Two new models are proposed using linear diffraction and the second-order part of the incident wave:

Moment-Based Model

- Eigenvalue analysis for moments
- Better prediction than standard linear theory
- Insufficient variability across field-points

Narrow-Band Model

- Closed form expression for extremes
- Better prediction of extremes than standard linear or moment-based model.
- Improved variability across field-points