

---

**NONLINEAR RANDOM OCEAN WAVES:  
PREDICTION AND COMPARISON WITH DATA**

**Steven R. Winterstein**  
*Stanford University*  
**Steven.Winterstein@stanford.edu**

**Alok K. Jha**  
*Risk Management Solutions, Inc.*  
**alokj@riskinc.com**

**Supported by**  
*Office of Naval Research*  
*Stanford Reliability of Marine Structures (RMS) Program*  
**Web Address: [www.stanford.edu/group/rms](http://www.stanford.edu/group/rms)**

---

# Motivation

---

**GOAL:** Probabilistic Models of Extreme Wave Crests

## WHY?

- Air gap—chance that wave hits deck  
(ISO: Set air gap to achieve target reliability)
- Floaters—chance that wave impacts hull; green water on deck
- General gross load (e.g., base shear) may correlate better with wave crest than height

## DATA SOURCES:

- Extensive wave tank measurements—multiple tests at  $T=100^+$  year return periods.
- Laser (and buoy) field data—here: 2 years at Ekofisk

## APPROACHES:

**Non-Gaussian Random Process Model:** 2nd-Order Random Stokes Waves (A.K. Jha Ph.D. thesis)

**Non-Rayleigh Random Variable Model of Crests:**  
“Noisy Stokes” model (leads to Weibull variable plus noise).

# References

---

- Jha, A. K. and S. R. Winterstein (2000). Nonlinear random ocean waves: Prediction and comparison with data. *Proc. 19th Intl. Offshore Mech. Arctic Eng. Symp.*, ASME, Paper No. OMAE00-6125.
- Sweetman, B. and S.R. Winterstein (1998). *Second-order random ocean waves: Prediction of temporal and spatial variation from fixed and moving references: the WAVEMAKER routine (Ver. 3.2)*, Rept. RMS-37.
- Jha, A.K. and S.R. Winterstein (1997). *Nonlinear random ocean waves: Prediction and comparison with data.* Rept-RMS-24.
- Jha, A. K. (1997). *Nonlinear stochastic models for ocean wave loads and responses of offshore structures and vessels.*, Stanford University.

# Introduction

---

- Dangers
  - deck impact
  - unusual dynamic response
- 2nd order models not common engineering practice
  - lack of convenient 2nd-order formulas
  - accuracy of 2nd-order models in question
  
- In this paper:
  - analytical formulas for wave moments (skewness, kurtosis)
  - probability distributions for wave elevation, crests, and heights
  - compare 2nd-order theory to wave tank and ocean wave measurements

# Wave Model

---

Total wave process:  $\eta(t) = \eta_1(t) + \eta_2(t)$

1st-order component:

$$\eta_1(t) = \sum_{k=1}^N A_k \cos(\omega_k t + \theta_k) = \operatorname{Re} \sum_{k=1}^N C_k \exp(i\omega_k t)$$

2nd-order component:

$$\eta_2(t) = \operatorname{Re} \sum_{m=1}^N \sum_{n=1}^N C_m C_n \left[ H_{mn}^+ e^{i(\omega_m + \omega_n)t} + H_{mn}^- e^{i(\omega_m - \omega_n)t} \right]$$

Second-order transfer function:  $H_{mn}^+ = f(\omega, k, d)$

Qualitative Notes:

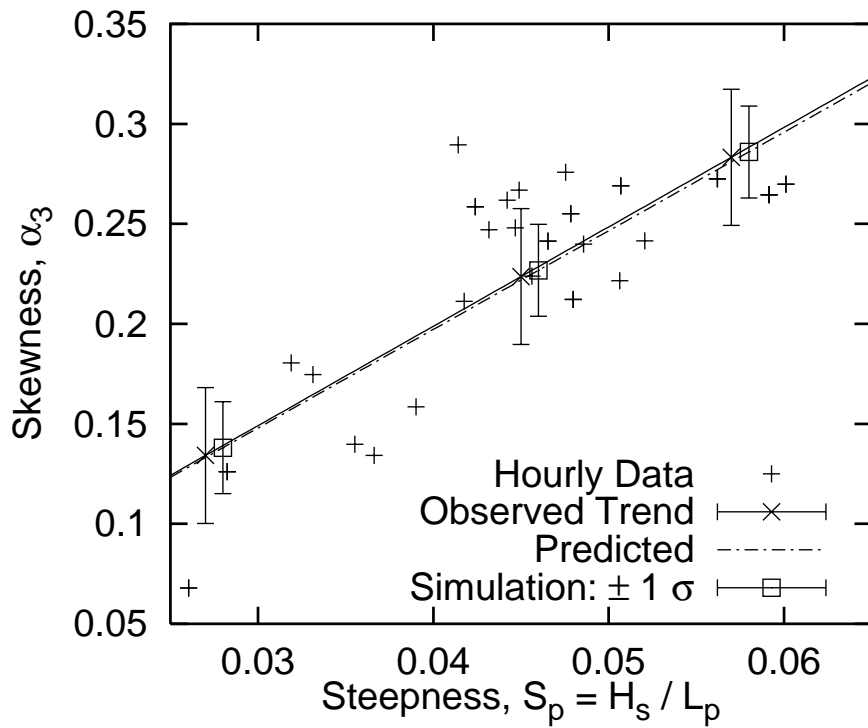
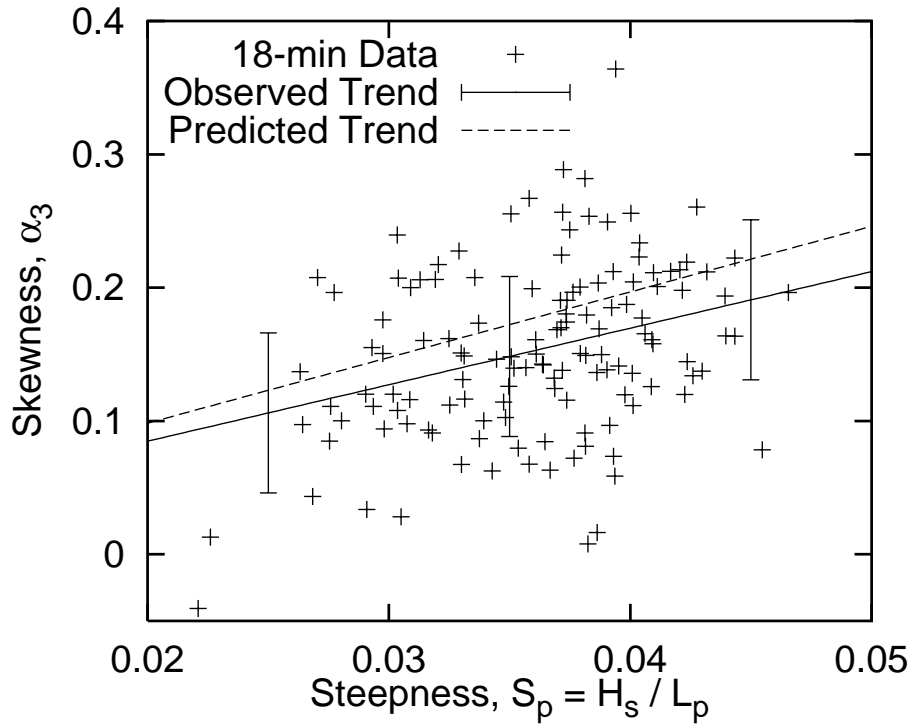
- more general than deterministic 2nd-order Stokes waves
- more general than sum of deterministic 2nd-order Stokes waves
- $\eta_1(t)$  preserves arbitrary wave spectrum (N components)
- result: for  $N = 10^3$  components,  $N^2 = 10^6$  components in  $\eta_2(t)$

# Wave Data Summary

---

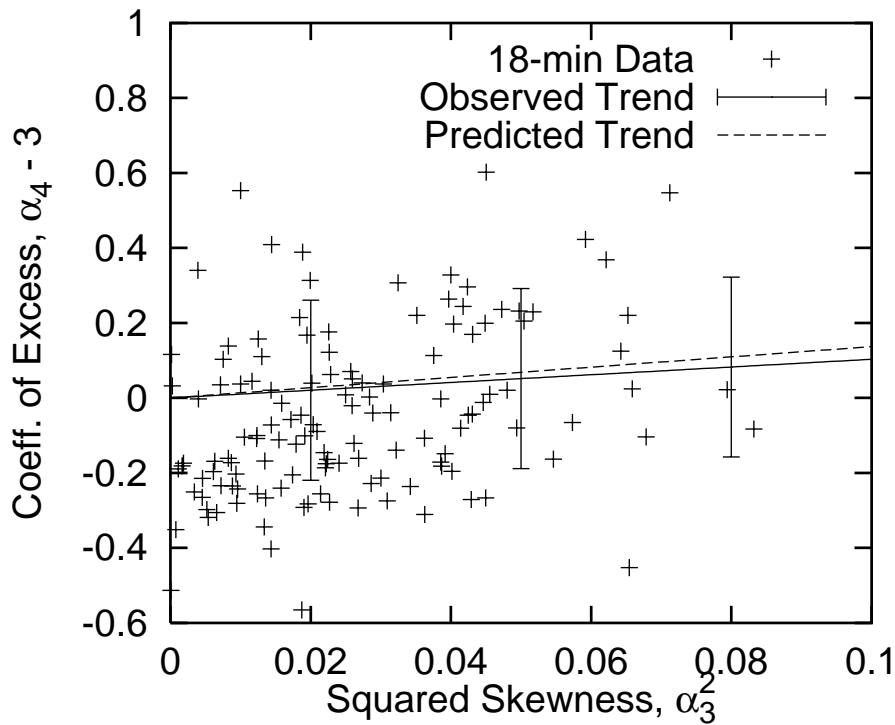
Description	$H_s$ (m)	$T_p$ (s)	Duration (hrs)
Snorre Wave Tank	13.4	13.75	5.79
Snorre (Set 2)	7.05	12.0	1.93
Ekofisk Field Data	5.14	9.8	3.98

# Skewness

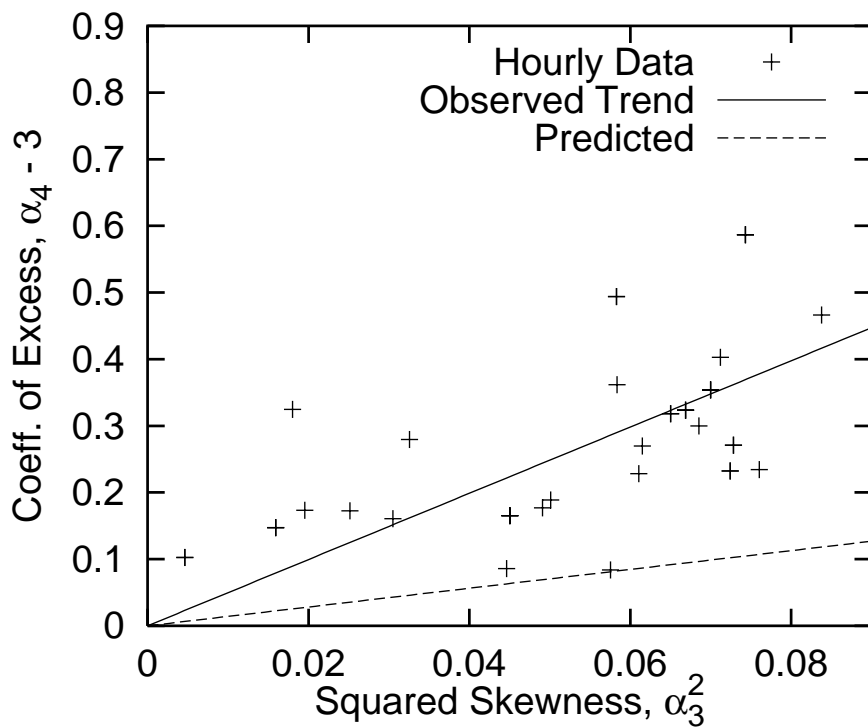


Conclusion: Both agree well

# Kurtosis



Field



WaveTank

Conclusion: Good agreement for field; wave tank underestimated

# 2nd Order Model vs. Analytical Models

---

Wave elevation model: (Hermite)

$$\text{Prob}[\text{Elevation} > \eta] = 1 - \Phi \left[ \frac{\eta - \bar{\eta}}{\sigma_\eta} \right]$$

$$x = g(u) = \bar{\eta} + \kappa\sigma_\eta \left[ u + \frac{\alpha_3}{6}(u^2 - 1) \right]$$

Crest Model: (Hermite)

$$\text{Prob}[C_r > c] = \exp(-0.5(c/\sigma_\eta)^2)$$

$$\eta_c = g(c_r) = \bar{\eta} + \kappa\sigma_\eta \left[ c_r + \frac{\alpha_3}{6}(c_r^2 - 1) \right]$$

Wave height model: (Naess)

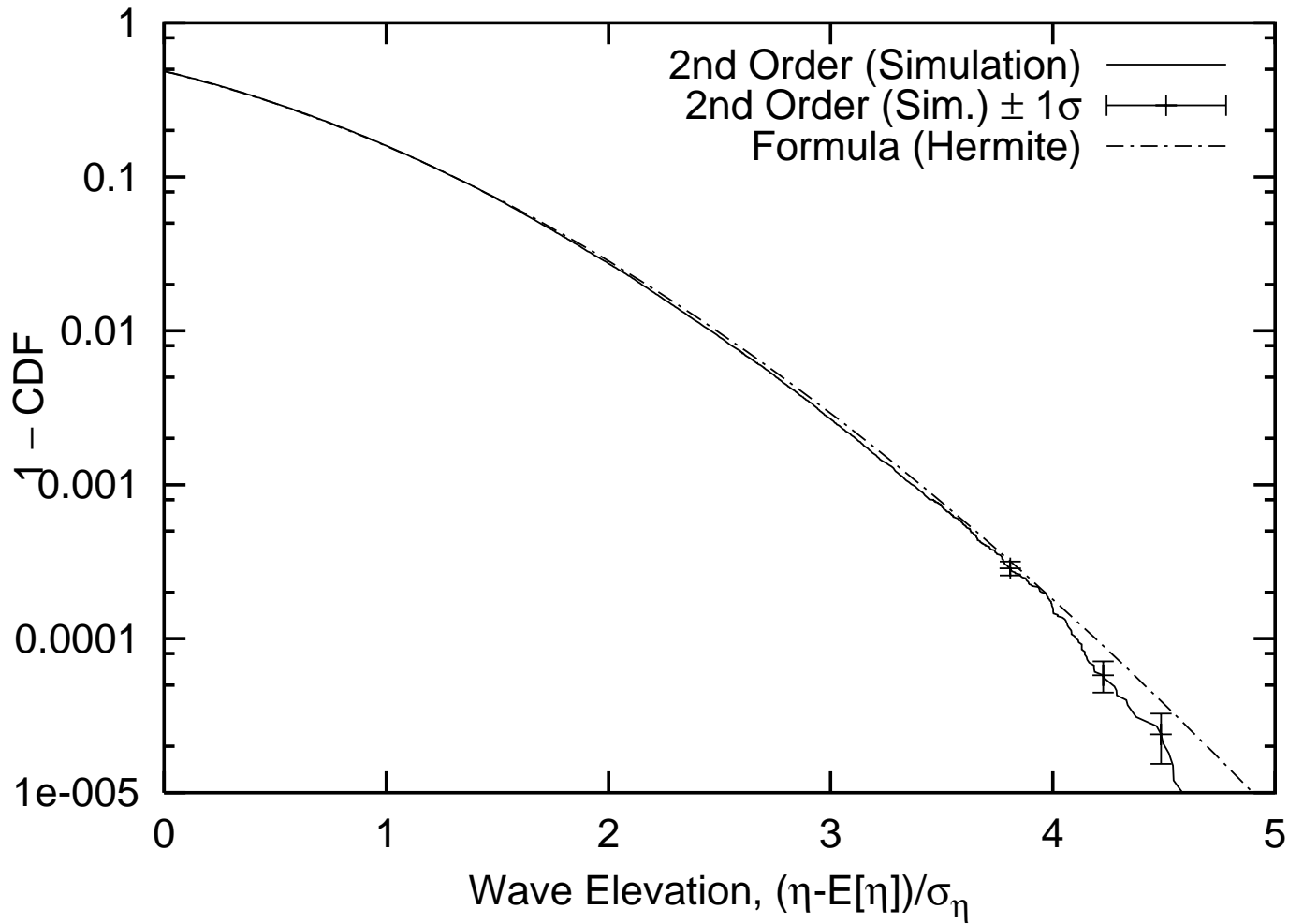
$$\text{Prob}[H_r > h_r] = \exp \left[ -\frac{(h_r/\sigma_\eta)^2}{8} \right]$$

$$\text{Prob}[H > h] = \exp \left[ -\frac{(h/\sigma_\eta)^2}{4(1 - \rho)} \right]$$

Note: Hermite here assumes kurtosis  $\approx 3$  (true for 2nd-order waves)

# Can simple formulas avoid simulation?

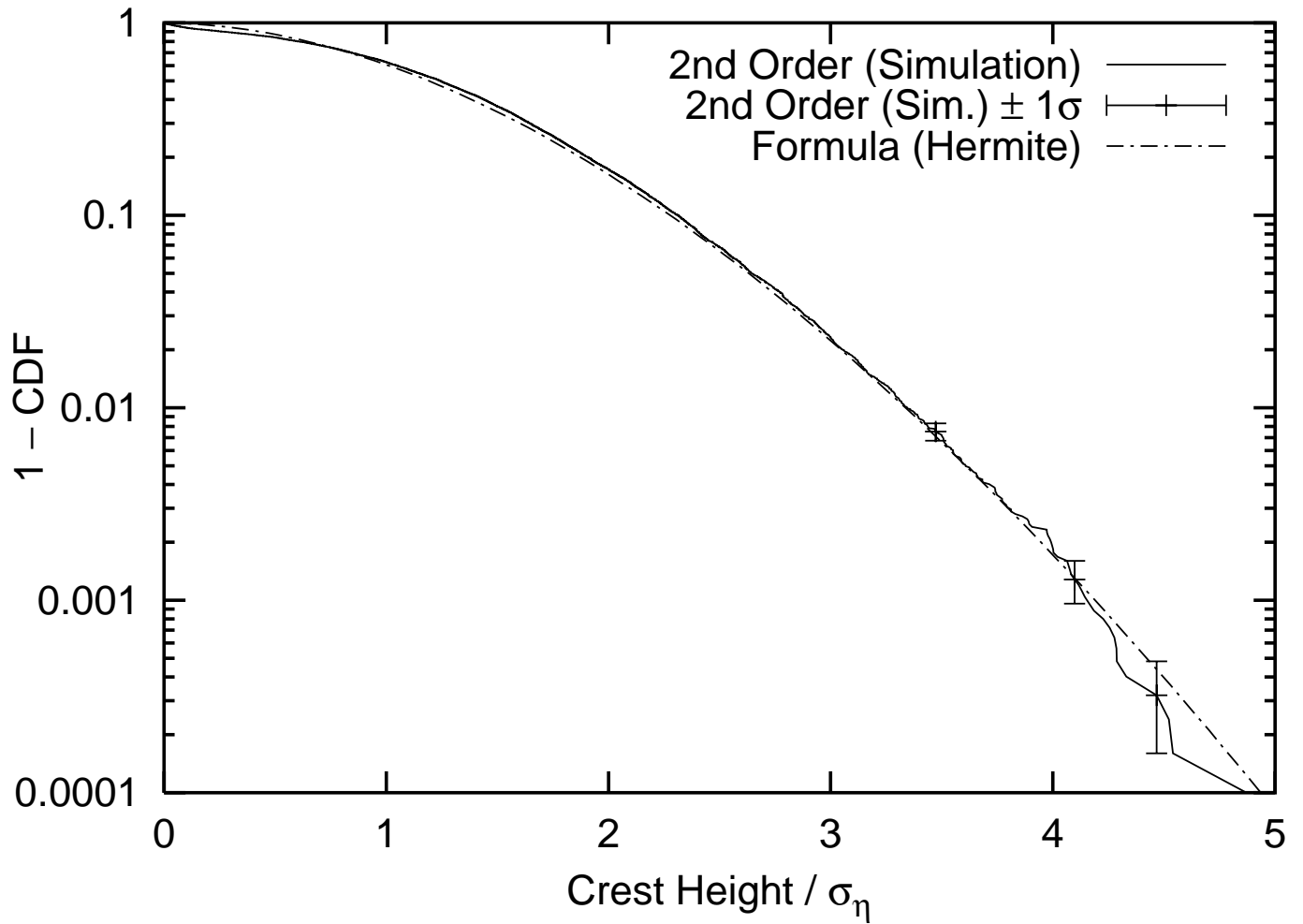
---



Wave elevation = Yes

# Can simple formulas avoid simulation?

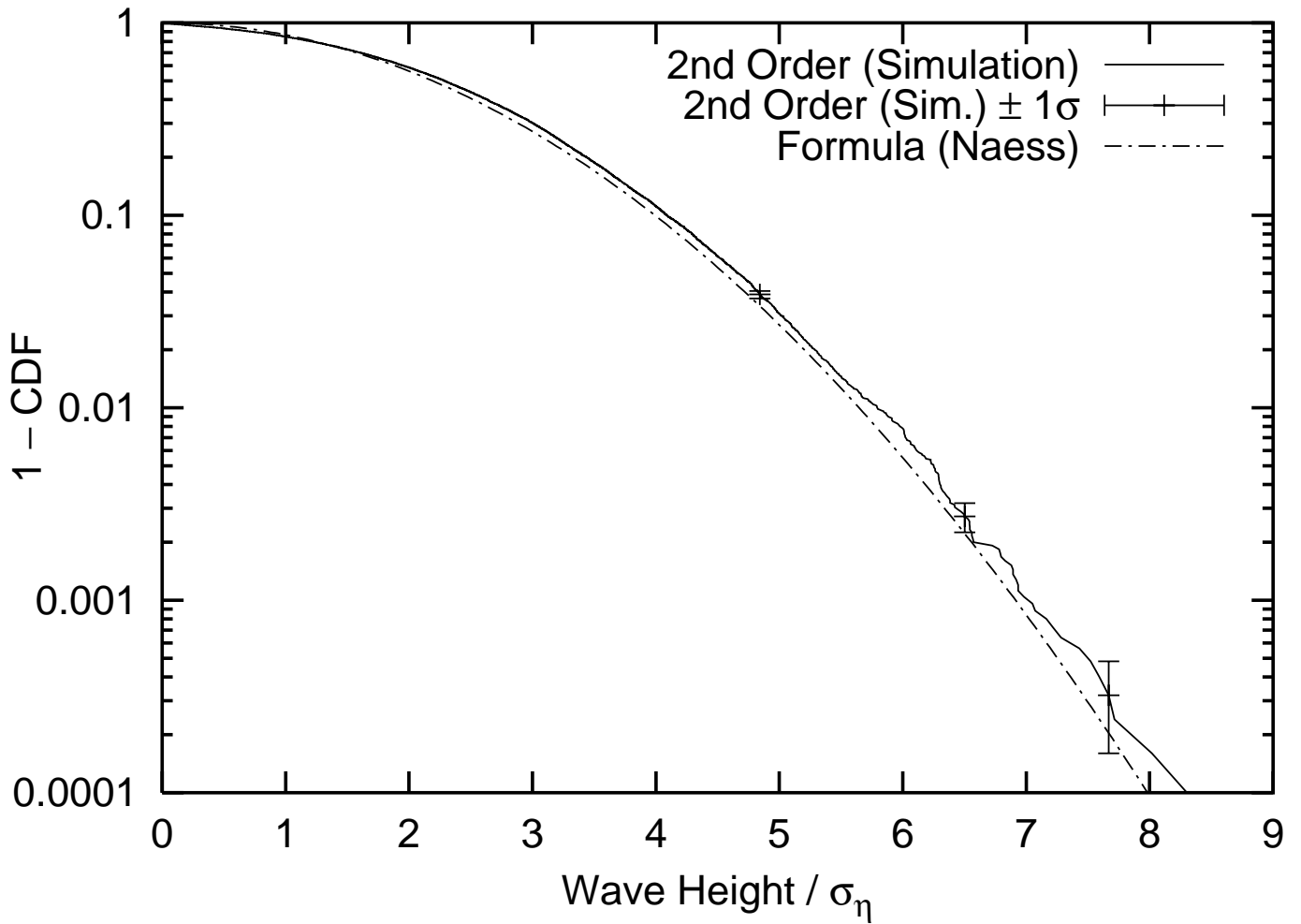
---



Wave Crest = Yes

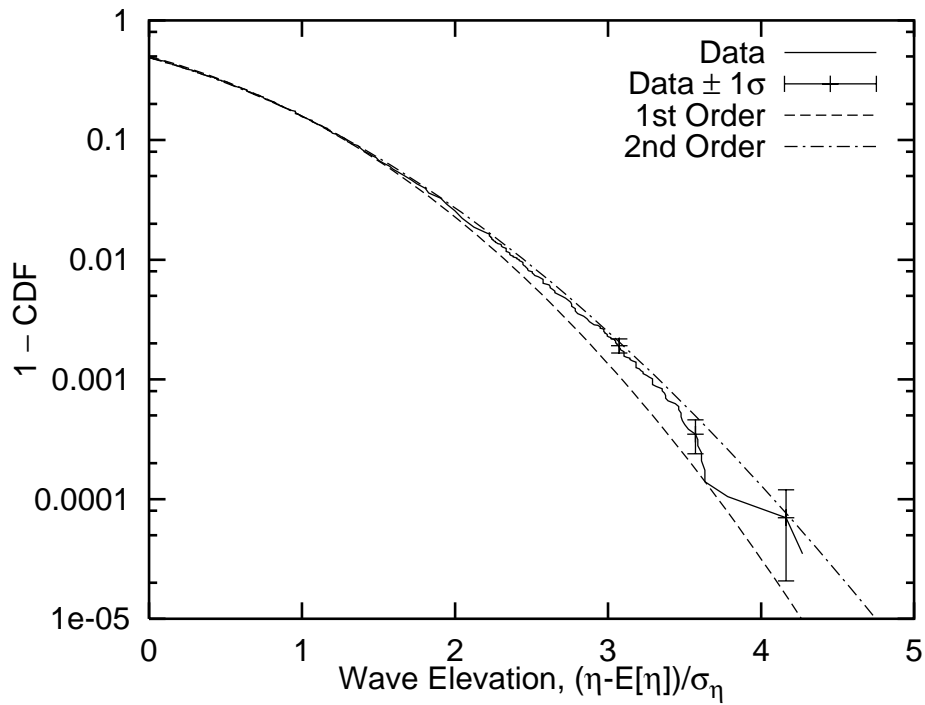
# Can simple formulas avoid simulation?

---

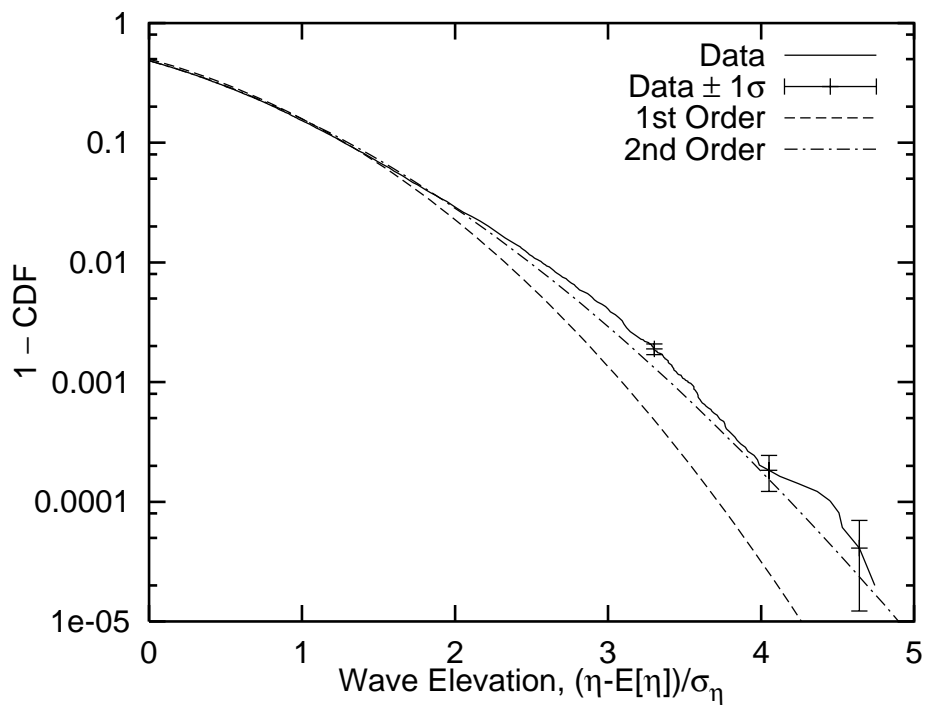


Wave Height = Yes

# Can 2nd-order model match data?



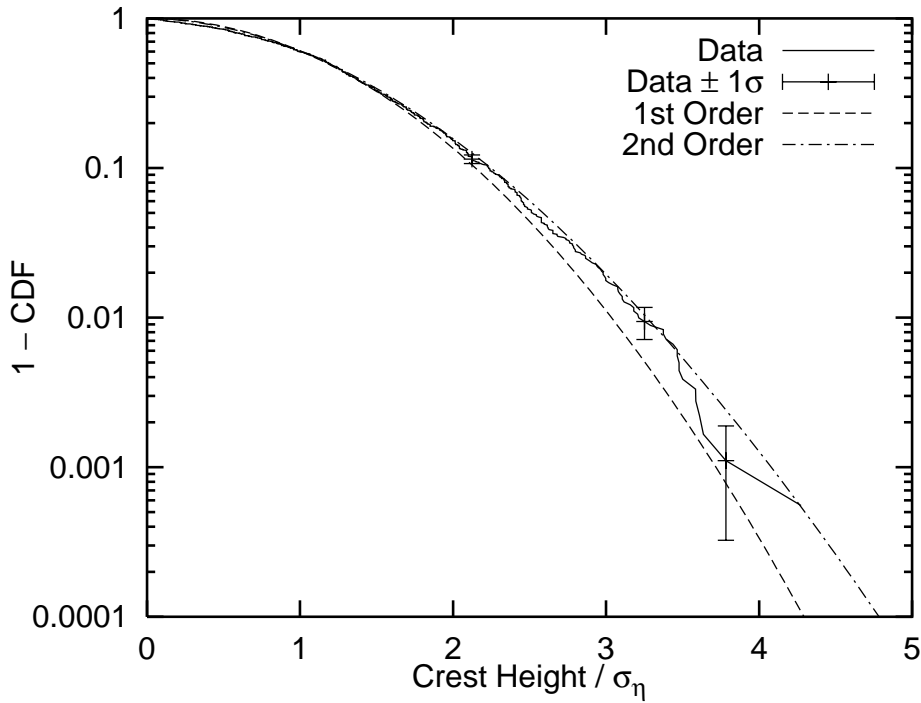
Field



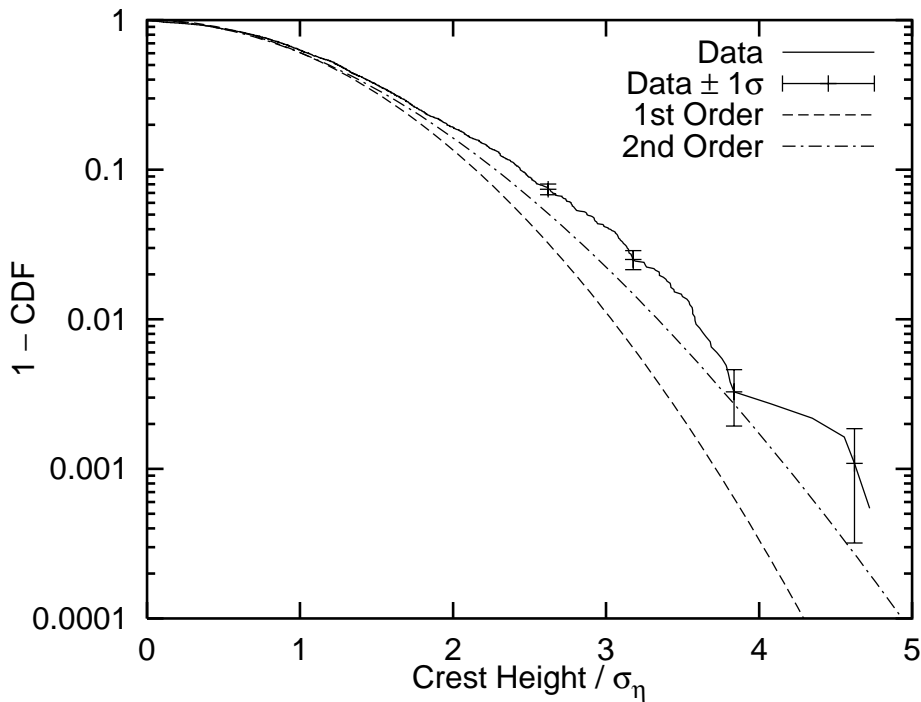
WaveTank

Wave Elevation: Field=Yes; WaveTank=Underestimated

# Can 2nd-order model match data?



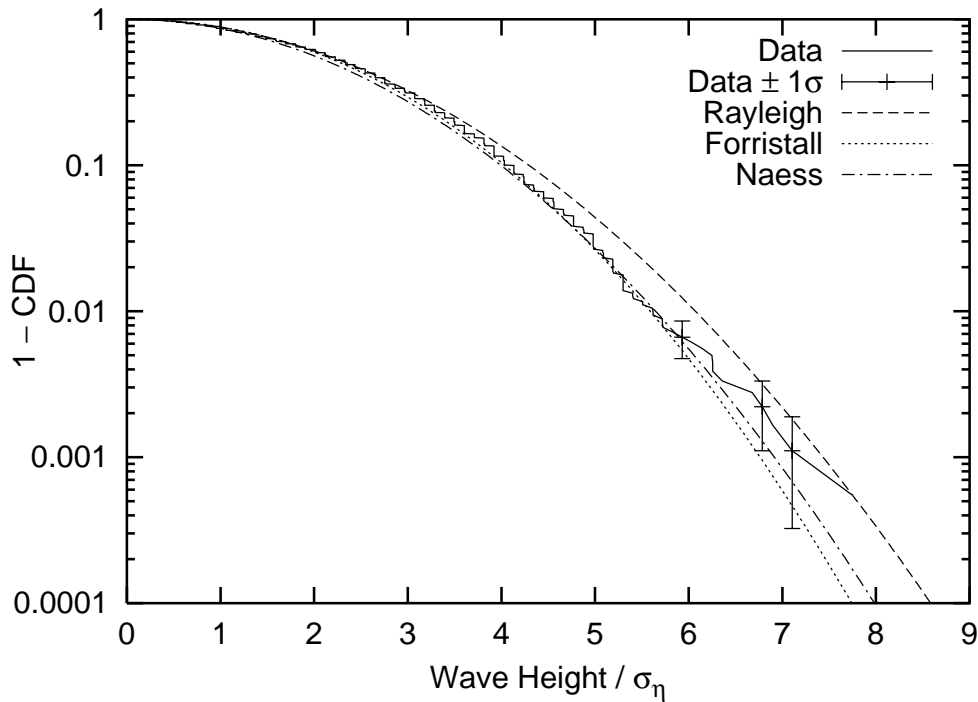
Field



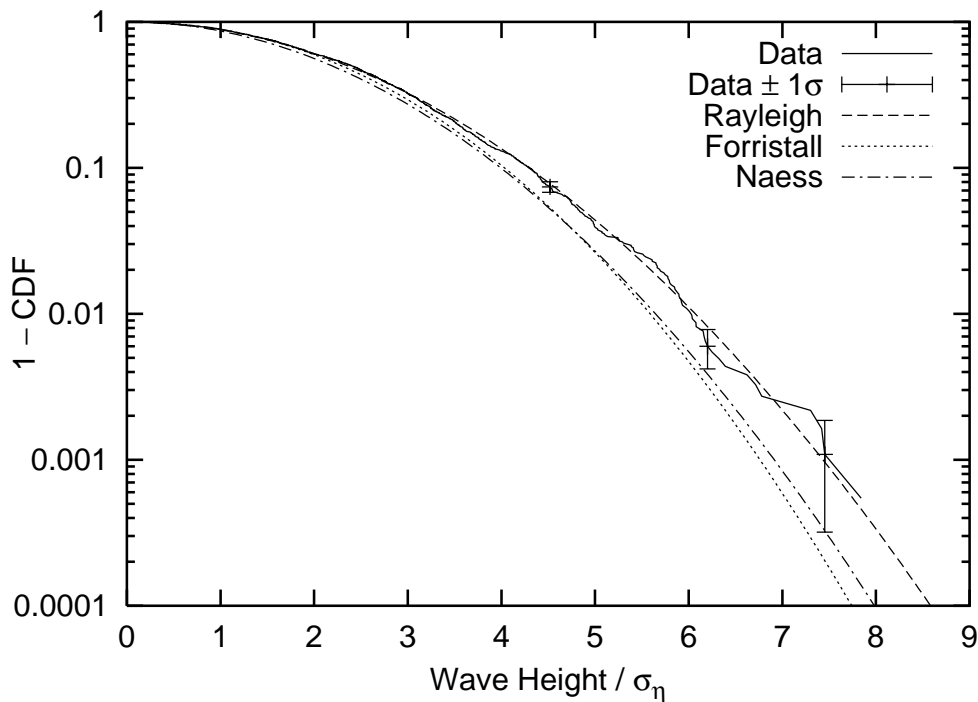
WaveTank

Wave Crest: Field=Yes; WaveTank=Underestimated

# Can 2nd-order model match data?



Field

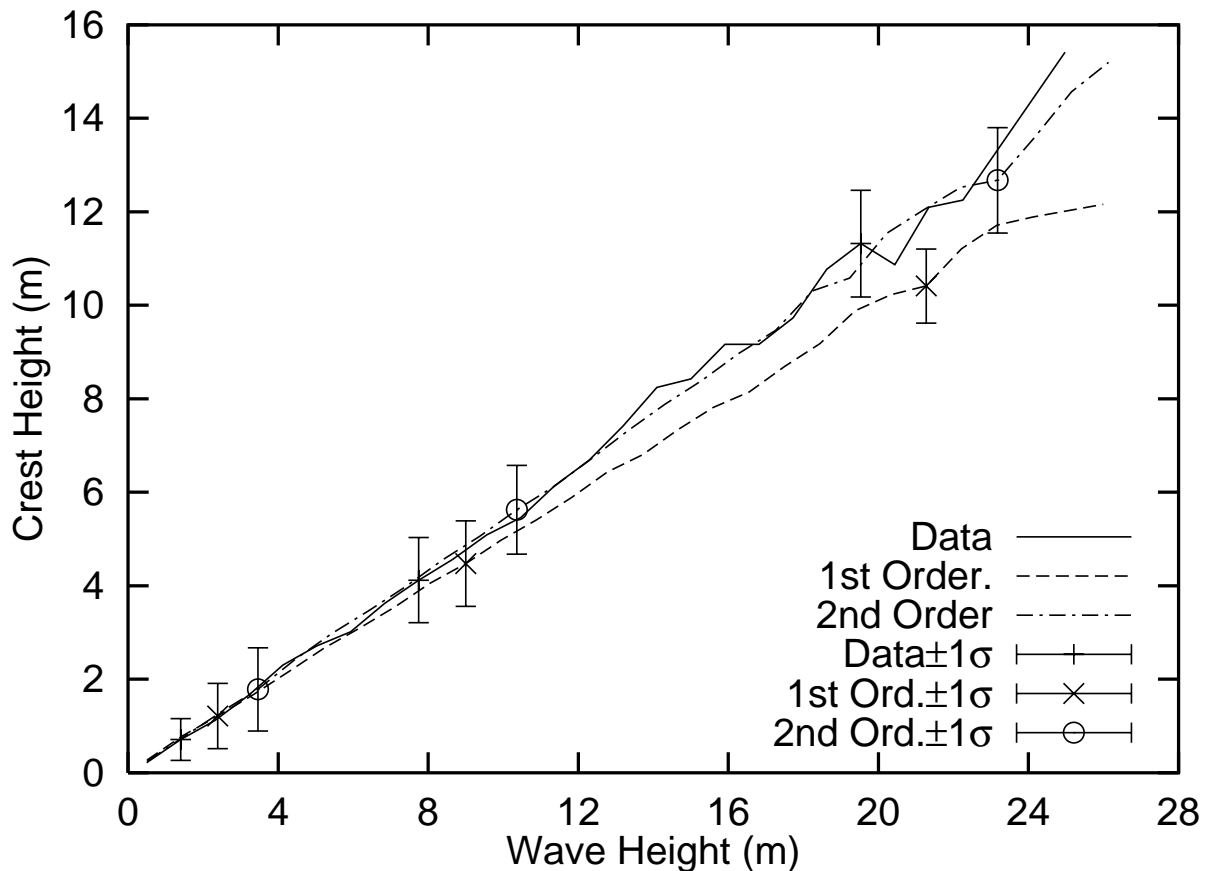


WaveTank

Wave Height: Field=Yes; WaveTank=Underestimated by best formulae (Naess; Forristall)

# Local Wave Statistics

---

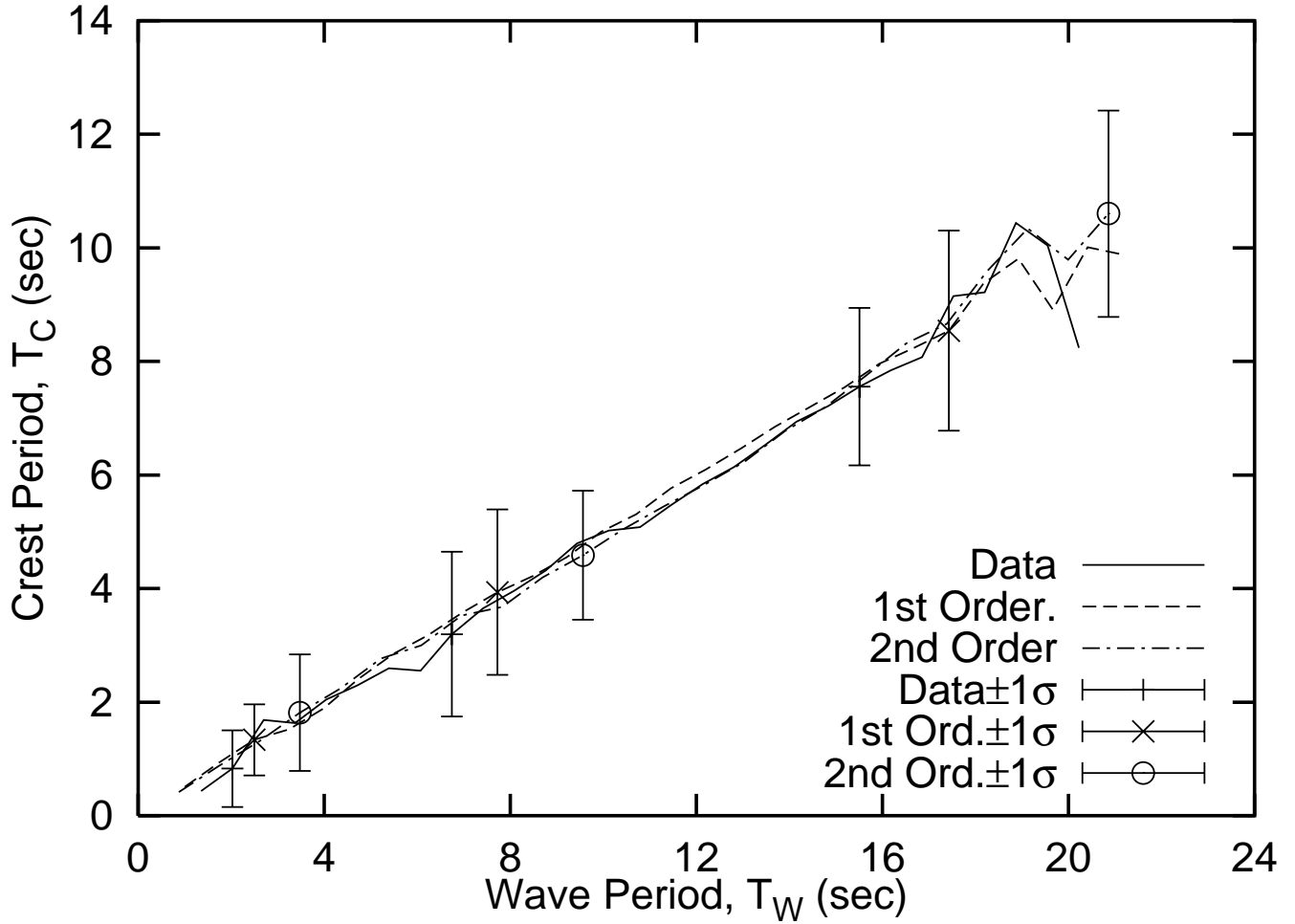


Notes: For extreme seas (wave tank tests)

- Both wave crest and height underestimated marginally
- Conditional crest given wave height accurate nonetheless
- For accurate crest height model:
  - Use Stokes theory for mean crest (given height); estimate crest variability from data (“Noisy Stokes”)
  - Use Hermite model with skewness from 2nd-order theory; estimate (enhanced) kurtosis from data (“Empirical Hermite”)

# Local Wave Statistics (contd)

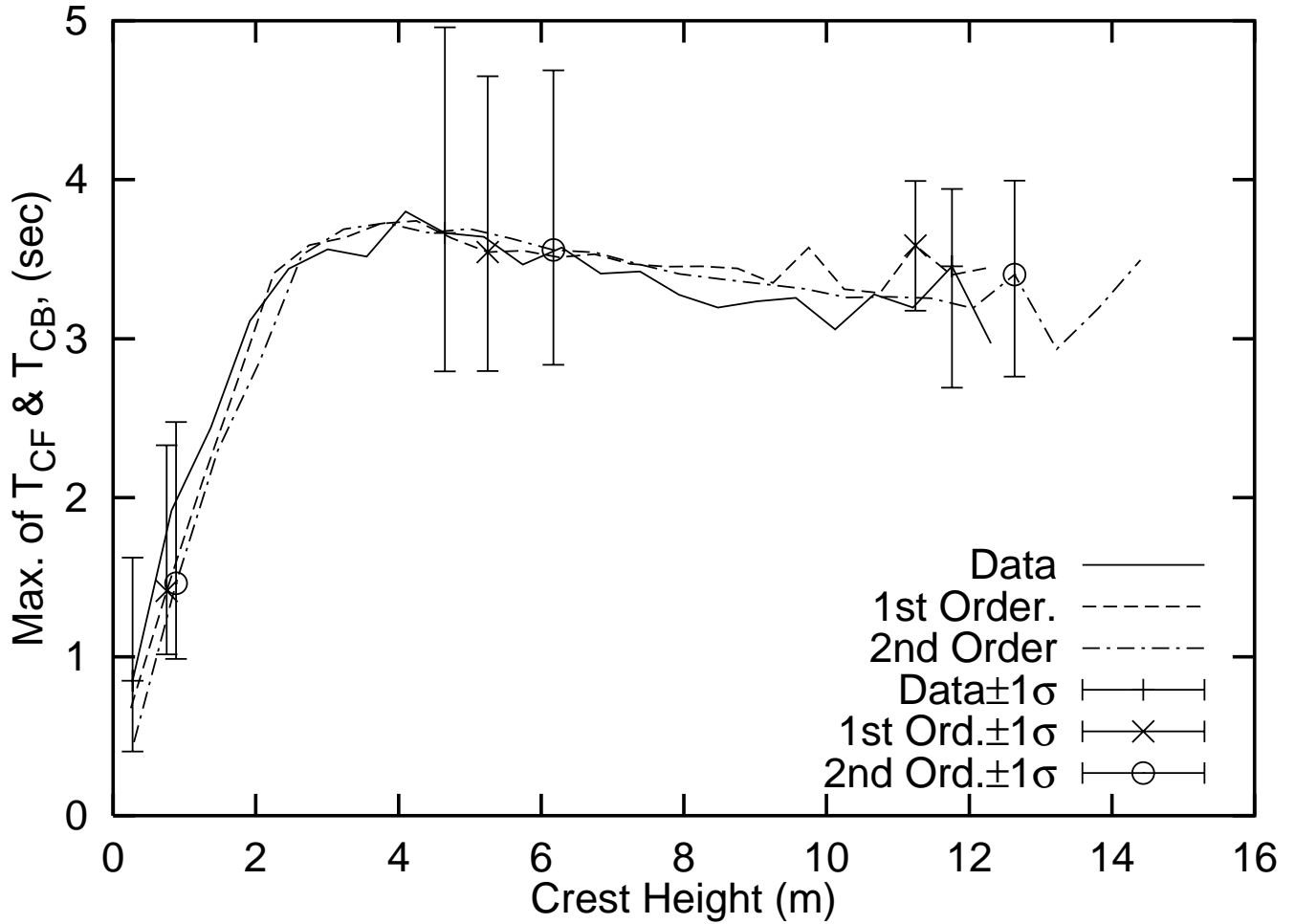
---



Crest Period  $T_C$  vs. wave period  $T_W$

# Local Wave Statistics (contd)

---



Maximum of crest front and crest back periods vs. crest height

# Conclusions

---

1. Convenient analytical expressions shown to match random 2nd-order waves

- wave skewness, kurtosis
- wave elevation/crest/height CDF

Hence: results for 2nd-order models available without simulation

2. Comparison between 2nd-order models and data:

- skewness:                   2nd-order model  $\approx$  field  $\approx$  wavetank data
- kurtosis:                   2nd-order model  $\approx$  field  $<$  wavetank data
- elevation/crest/height:  
                                  2nd-order model  $\approx$  field  $<$  wavetank data
- Profile shapes (C/H, steepness) accurately predicted

**Recommendations:** For accurate crest height model

- Use Stokes theory for mean crest (given height); estimate crest variability from data (“Noisy Stokes”)
- Use Hermite model with skewness from 2nd-order theory; estimate (enhanced) kurtosis from data (“Empirical Hermite”)