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NONLINEAR RANDOM OCEAN WAVES: PREDICTION AND COMPARISON WITH DATA

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Motivation

GOAL: Probabilistic Models of Extreme Wave Crests

WHY?

- Air gap—chance that wave hits deck (ISO: Set air gap to achieve target reliability)
- Floaters—chance that wave impacts hull; green water on deck
- General gross load (e.g., base shear) may correlate better with wave crest than height

DATA SOURCES:

- Extensive wave tank measurements—multiple tests at T=100$^+$ year return periods.
- Laser (and buoy) field data—here: 2 years at Ekofisk

APPROACHES:

Non-Gaussian Random Process Model: 2nd-Order Random Stokes Waves (A.K. Jha Ph.D. thesis)

Non-Rayleigh Random Variable Model of Crests: "Noisy Stokes" model (leads to Weibull variable plus noise).

Introduction

- Dangers
  - deck impact
  - unusual dynamic response
- 2nd order models not common engineering practice
  - lack of convenient 2nd-order formulas
  - accuracy of 2nd-order models in question

- In this paper:
  - analytical formulas for wave moments (skewness, kurtosis)
  - probability distributions for wave elevation, crests, and heights
  - compare 2nd-order theory to wave tank and ocean wave measurements

Wave Model

Total wave process: $\eta(t) = \eta_1(t) + \eta_2(t)$

1st-order component:

$$\eta_1(t) = \sum_{k=1}^{N} A_k \cos(\omega_k t + \theta_k) = \mathrm{Re} \sum_{k=1}^{N} C_k \exp(i\omega_k t)$$

2nd-order component:

$$\eta_2(t) = \mathrm{Re} \sum_{m=1}^{N} \sum_{n=1}^{N} C_m C_n \left[H_{mn}^{+} \mathrm{e}^{i(\omega_m + \omega_n)t} + H_{mn}^{-} \mathrm{e}^{i(\omega_m - \omega_n)t} \right]$$

Second-order transfer function: $H_{mn}^{+} = f(w, k, d)$

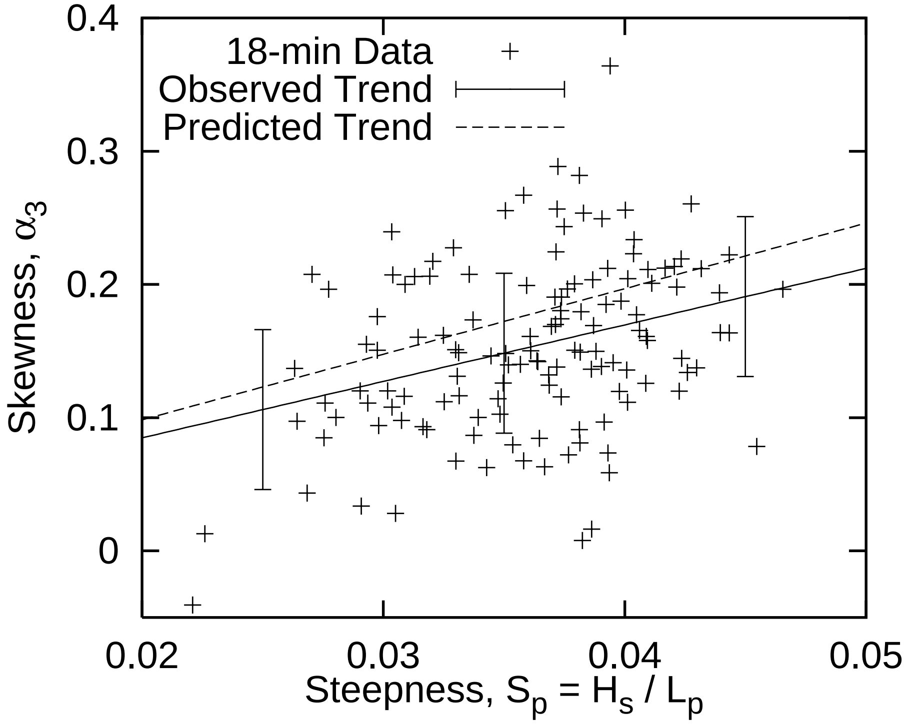
Qualitative Notes:

- more general than deterministic 2nd-order Stokes waves
- more general than sum of deterministic 2nd-order Stokes waves
- $\eta_1(t)$ preserves arbitrary wave spectrum (N components)
- result: for $N = 10^3$ components, $N^2 = 10^6$ components in $\eta_2(t)$

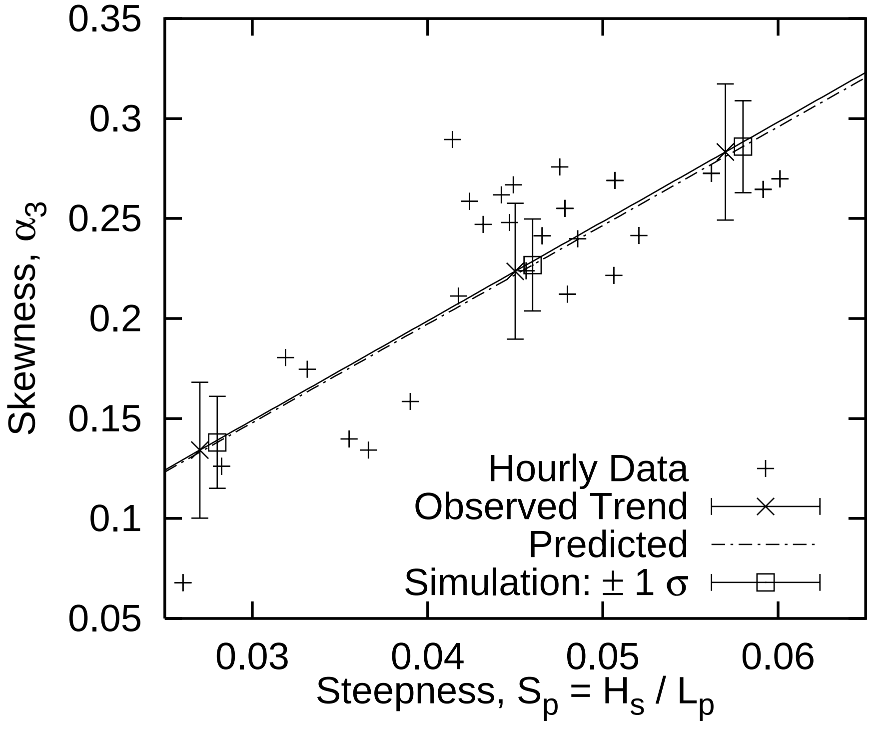
Wave Data Summary

Description	H_s(m)	T_p(s)	Duration (hrs)
Snorre Wave Tank	13.4	13.75	5.79
Snorre (Set 2)	7.05	12.0	1.93
Ekofisk Field Data	5.14	9.8	3.98

Skewness



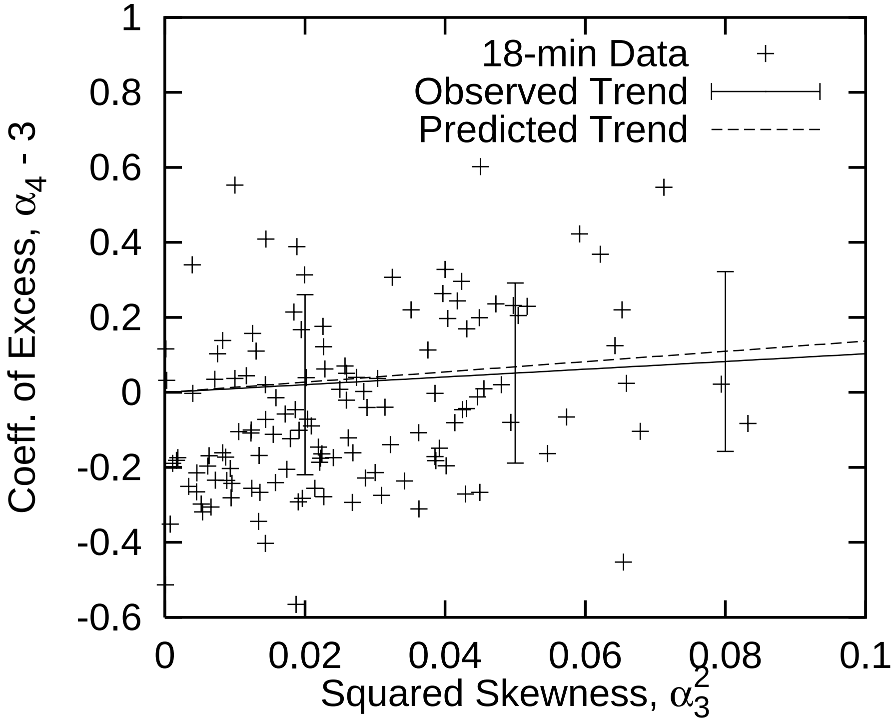
Field



WaveTank

Conclusion: Both agree well

Kurtosis



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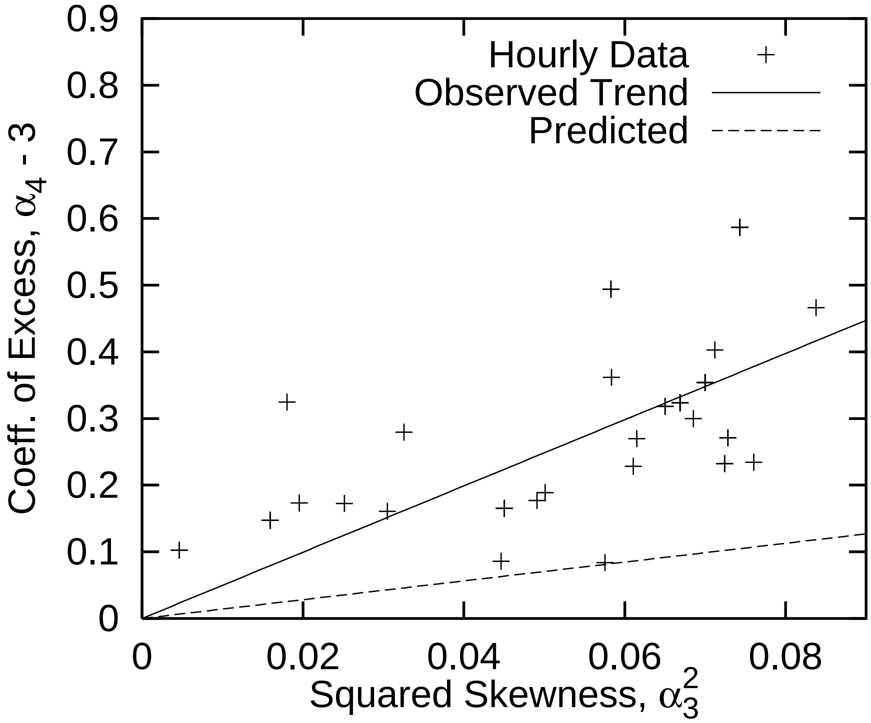


WaveTank

Conclusion: Good agreement for field; wave tank underestimated

2nd Order Model vs. Analytical Models

Wave elevation model: (Hermite)

$$\text{Prob}[\text{Elevation} > \eta] = 1 - \Phi\left[\frac{\eta - \overline{\eta}}{\sigma_\eta}\right]$$

$$x = g(u) = \overline{\eta} + \kappa\sigma_\eta\left[u + \frac{\alpha_3}{6}(u^2 - 1)\right]$$

Crest Model: (Hermite)

$$\text{Prob}[C_r > c] = \exp(-0.5(c/\sigma_\eta)^2)$$

$$\eta_c = g(c_r) = \overline{\eta} + \kappa\sigma_\eta\left[c_r + \frac{\alpha_3}{6}(c_r^2 - 1)\right]$$

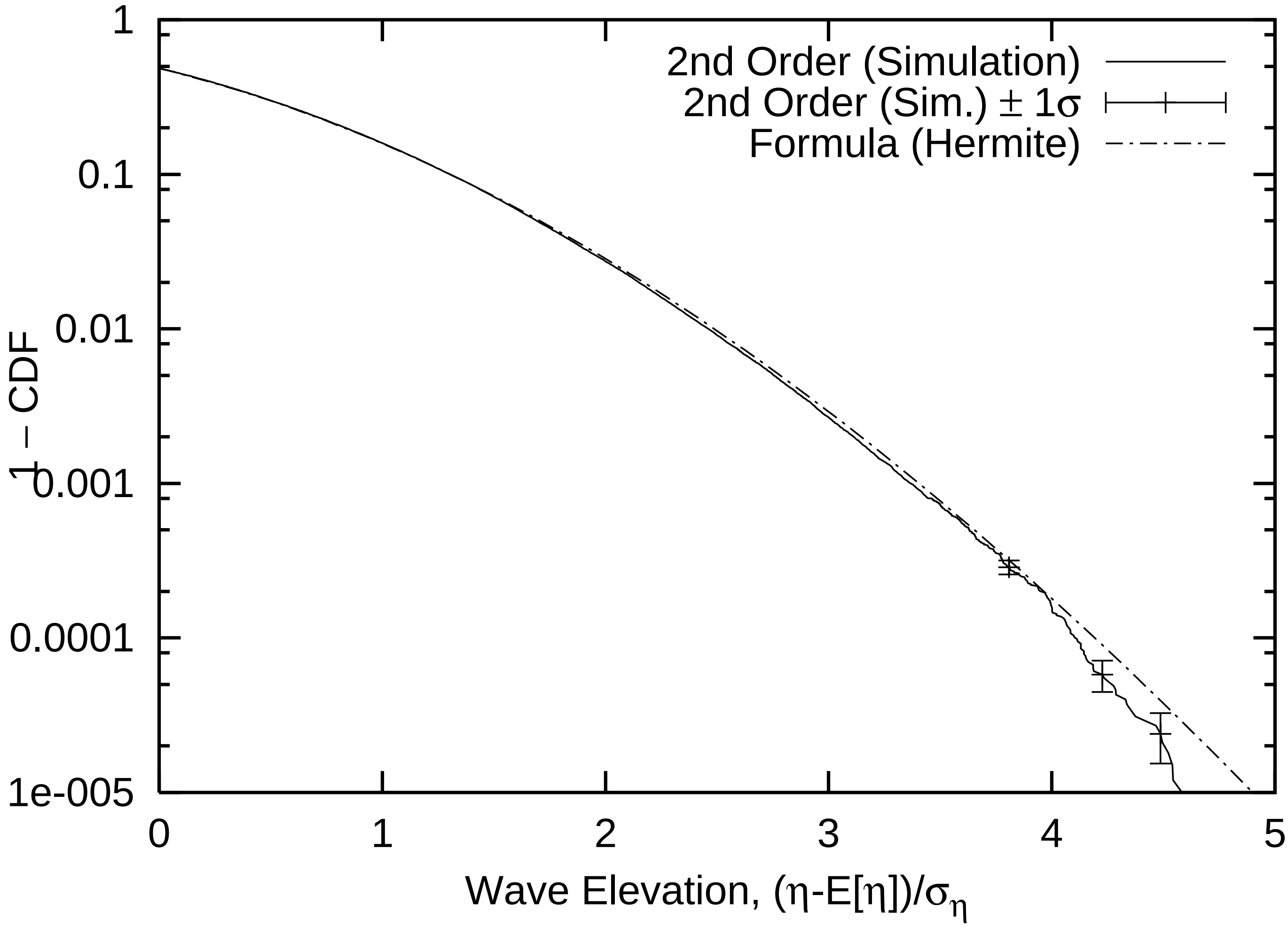
Wave height model: (Naess)

$$\text{Prob}[H_r > h_r] = \exp\left[-\frac{(h_r/\sigma_\eta)^2}{8}\right]$$

$$\text{Prob}[H > h] = \exp\left[-\frac{(h/\sigma_\eta)^2}{4\,(1-\rho)}\right]$$

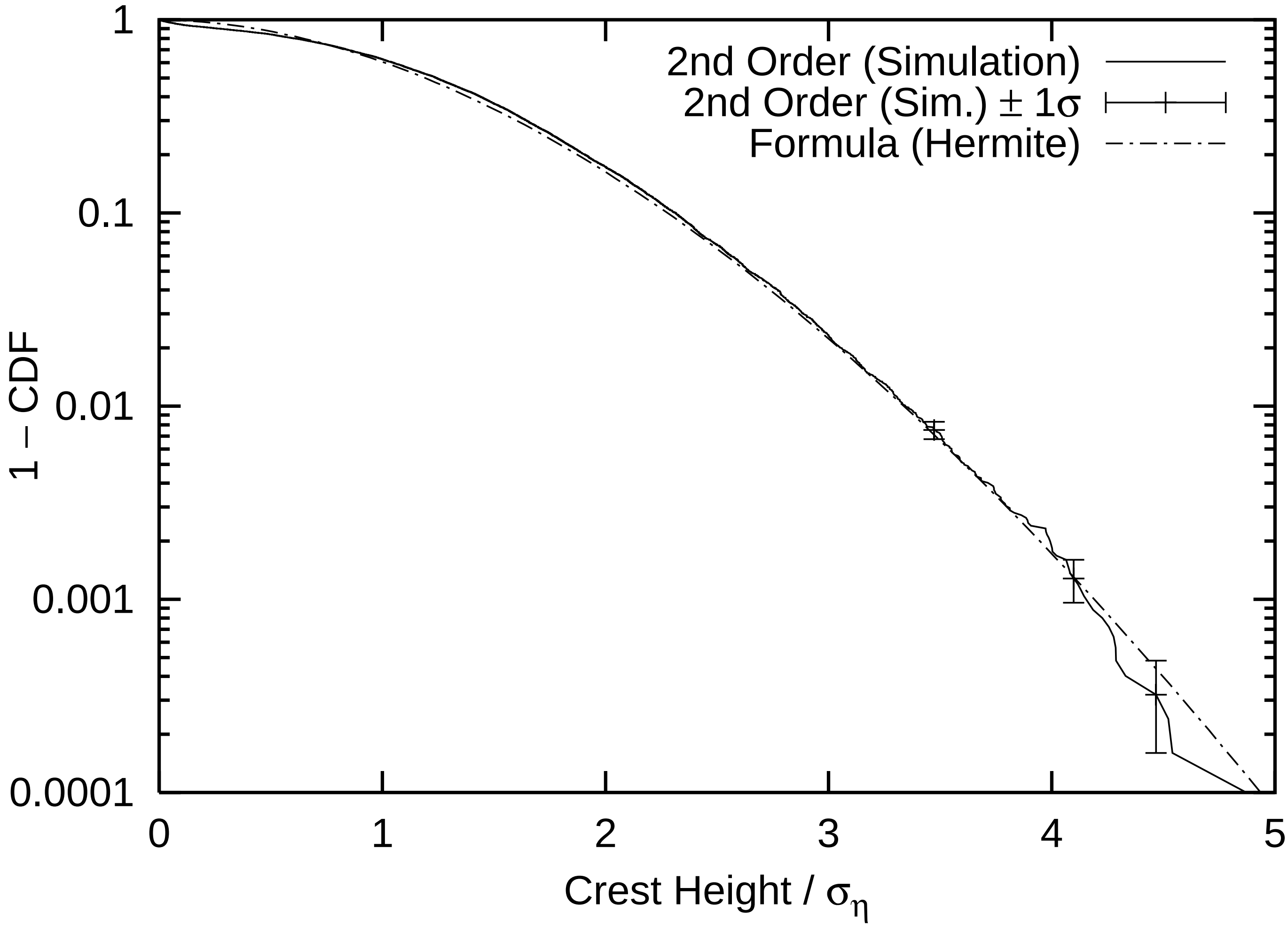
Note: Hermite here assumes kurtosis≈3 (true for 2nd-order waves)

Can simple formulas avoid simulation?



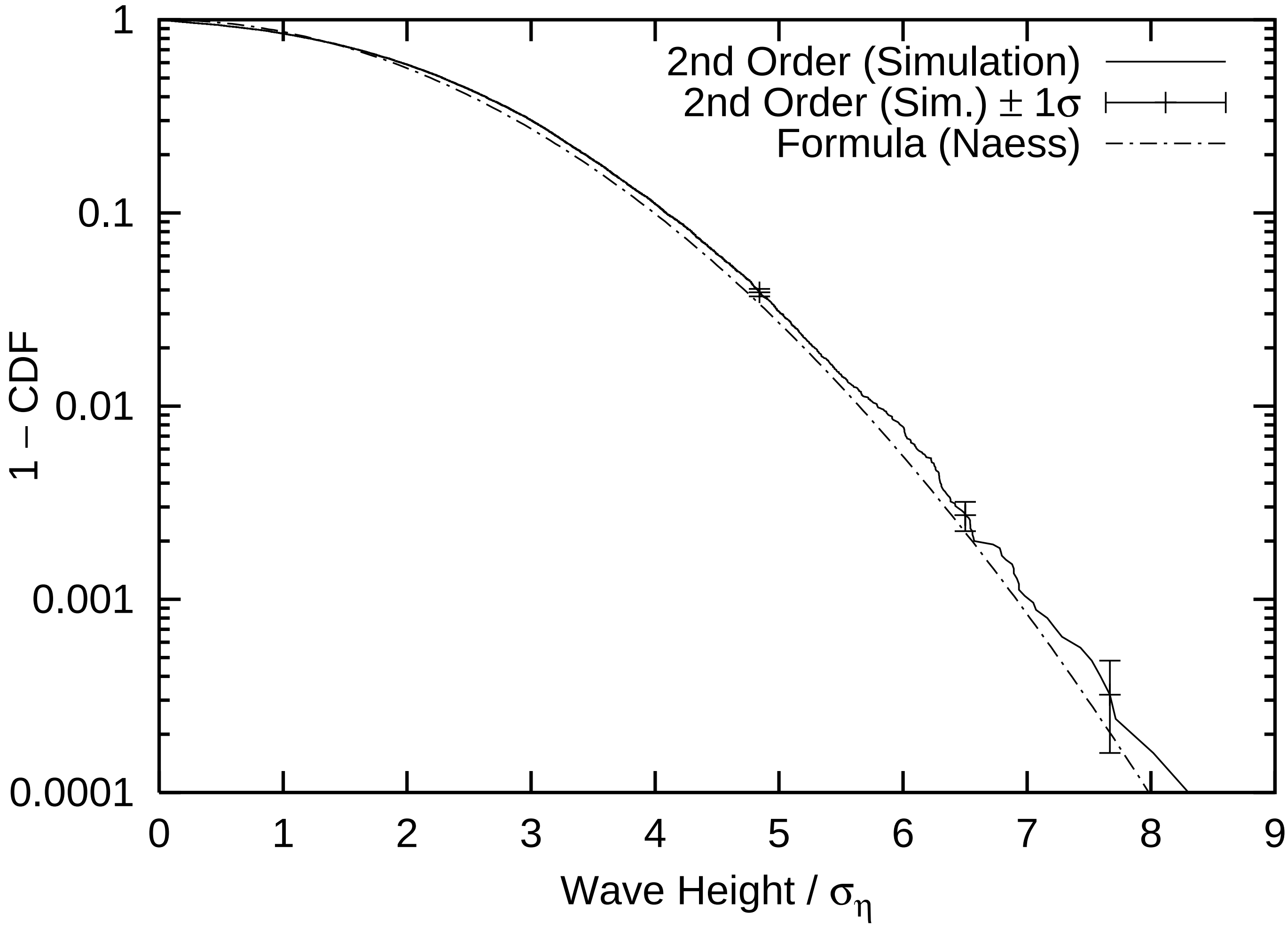
Wave elevation = Yes

Can simple formulas avoid simulation?



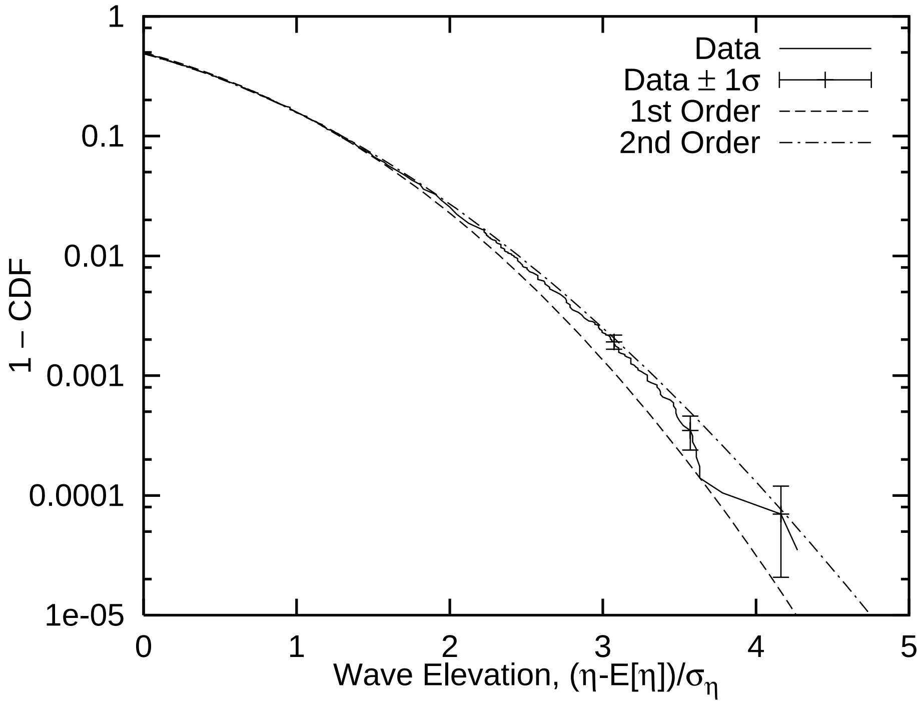
Wave Crest = Yes

Can simple formulas avoid simulation?



Wave Height = Yes

Can 2nd-order model match data?



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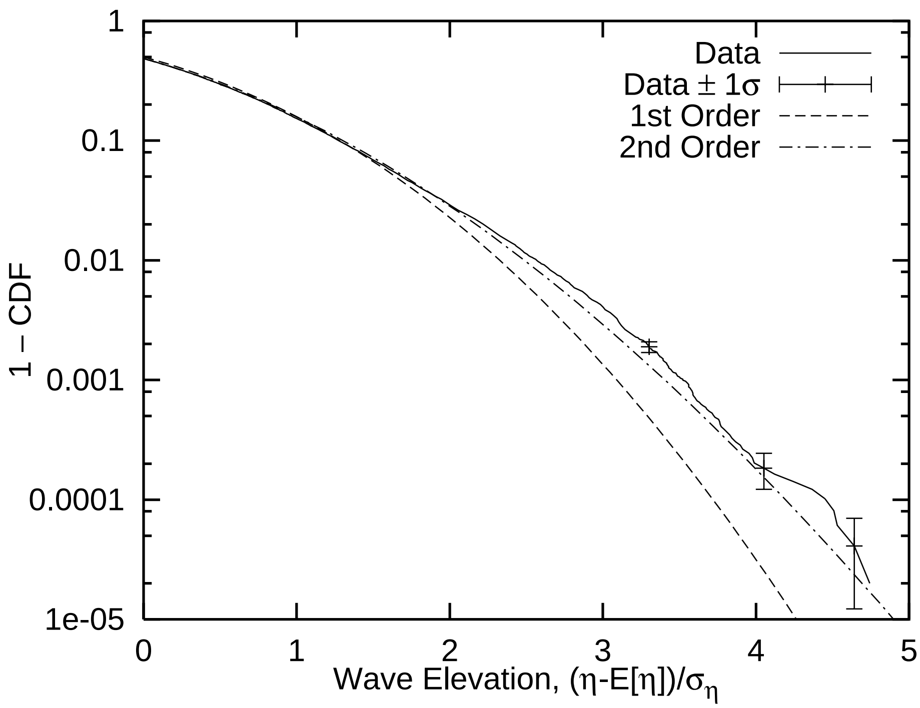
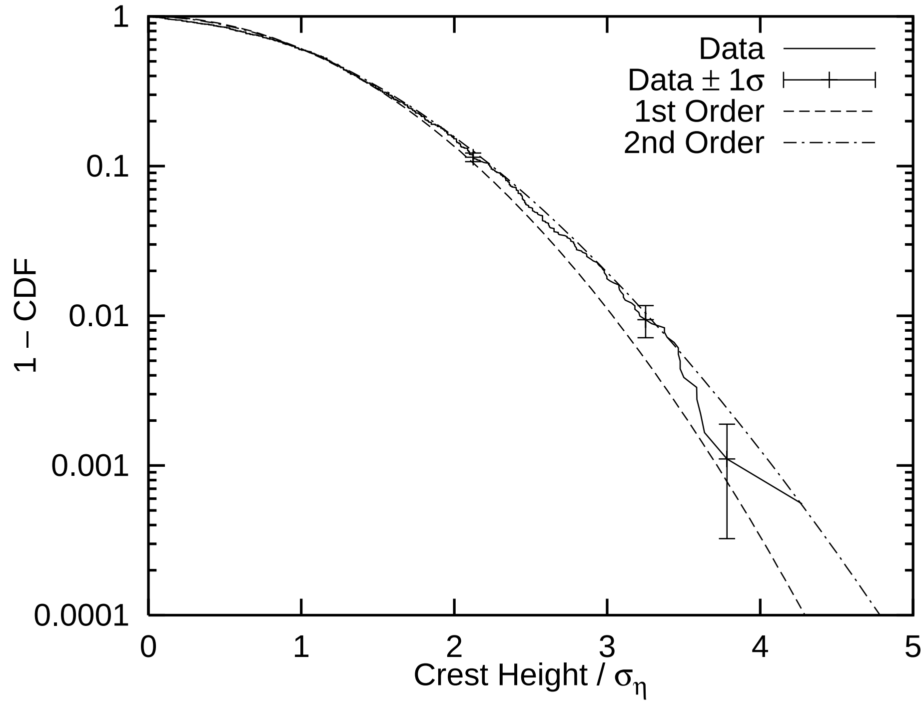


WaveTank

Wave Elevation: Field=Yes; WaveTank=Underestimated

Can 2nd-order model match data?



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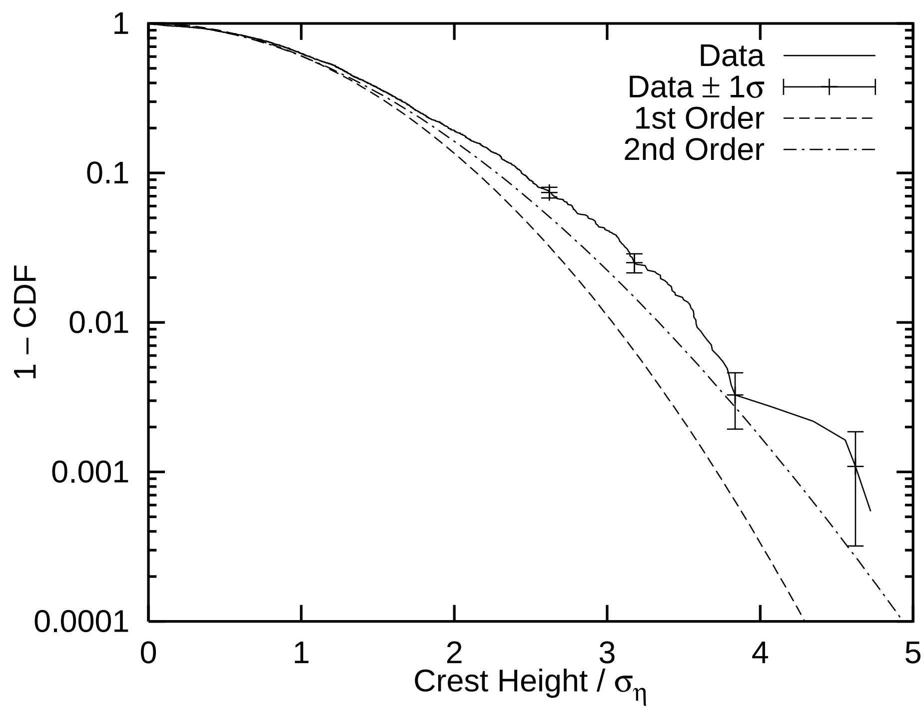
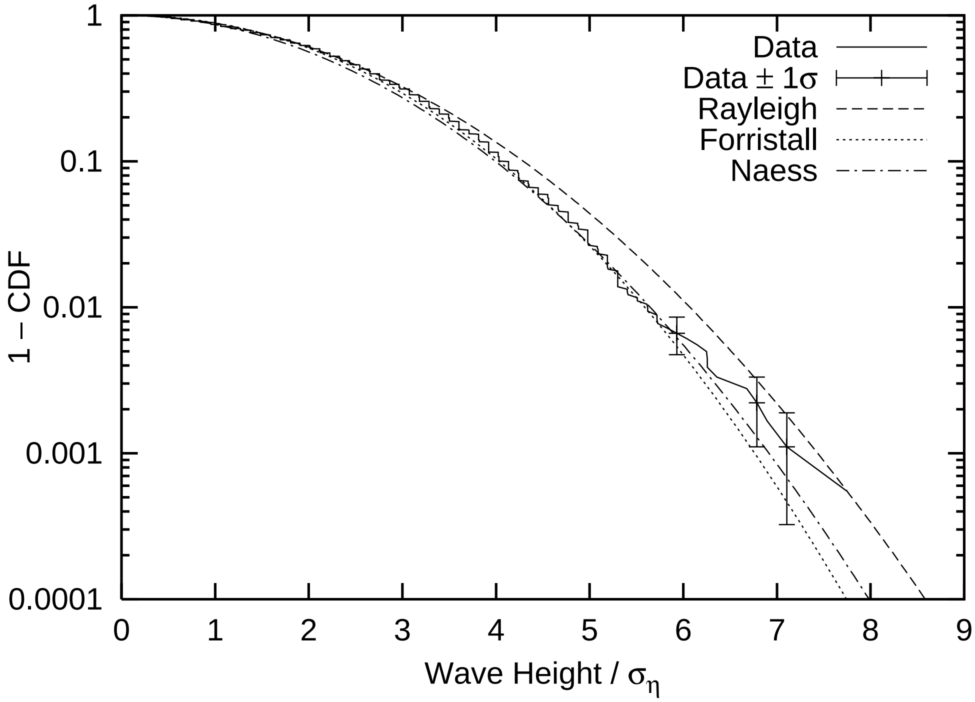


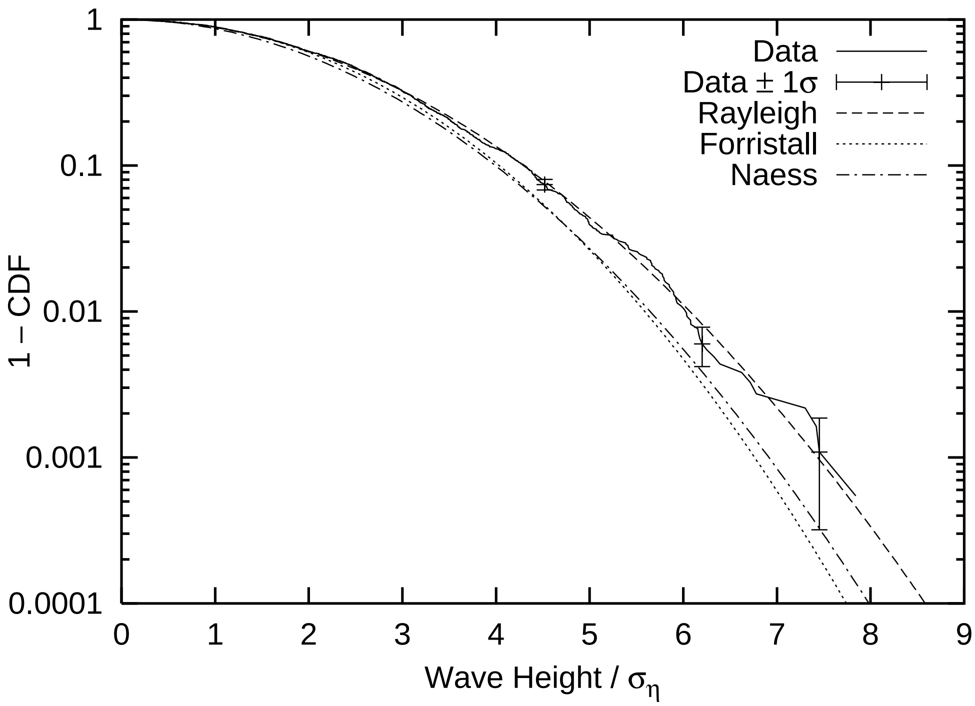
WaveTank

Wave Crest: Field=Yes; WaveTank=Underestimated

Can 2nd-order model match data?



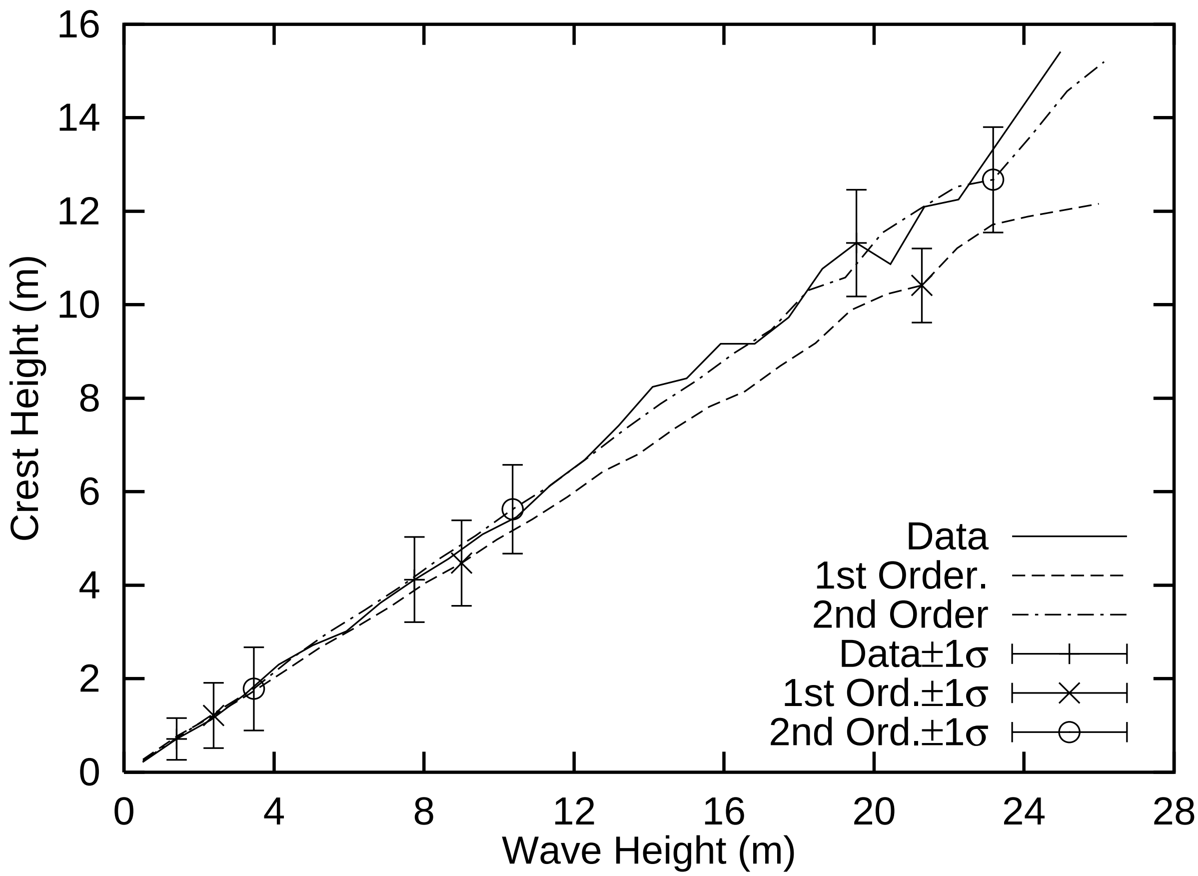
Field



WaveTank

Wave Height: Field=Yes; WaveTank=Underestimated by best formulae (Naess; Forristall)

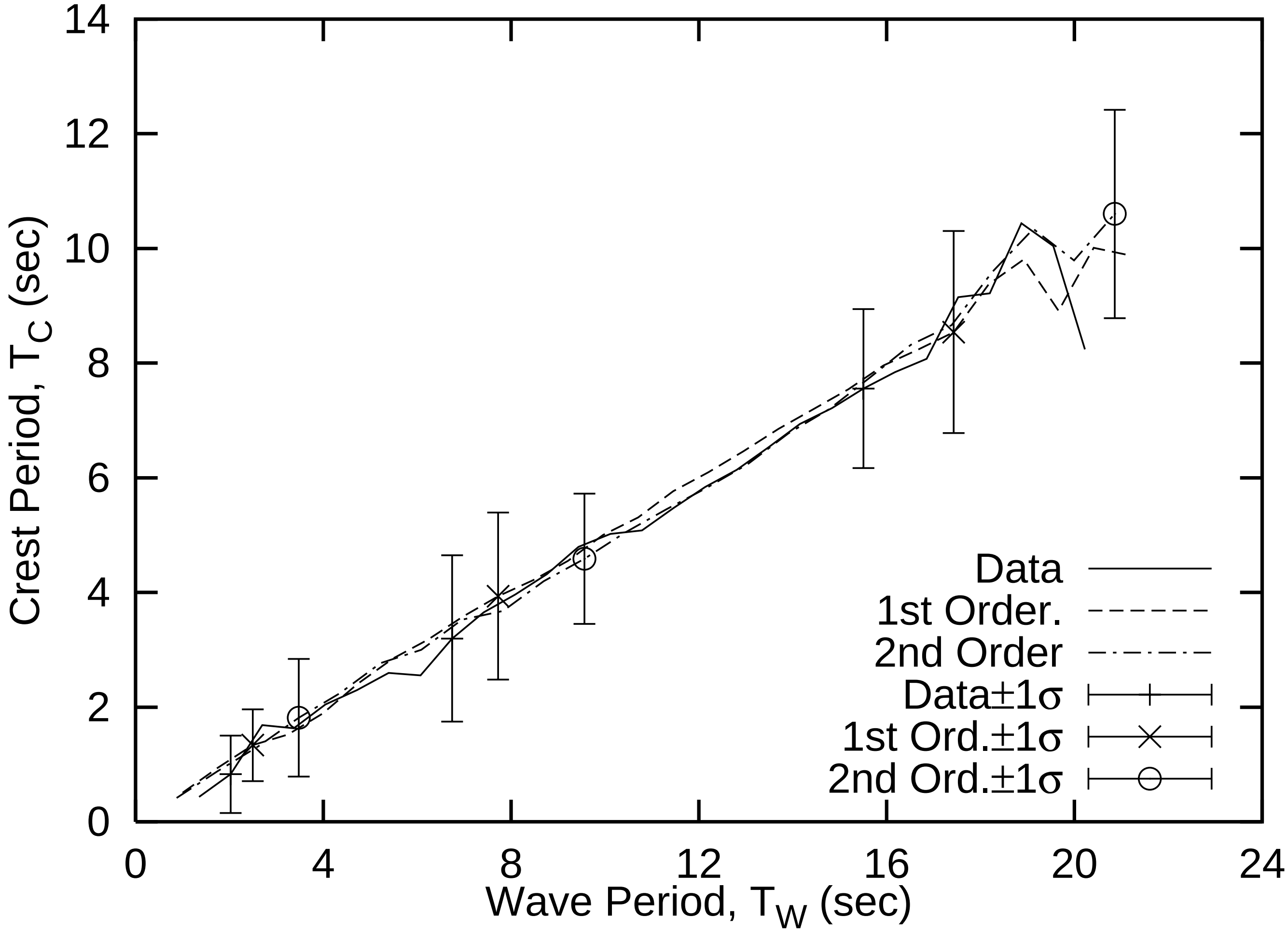
Local Wave Statistics



Notes: For extreme seas (wave tank tests)

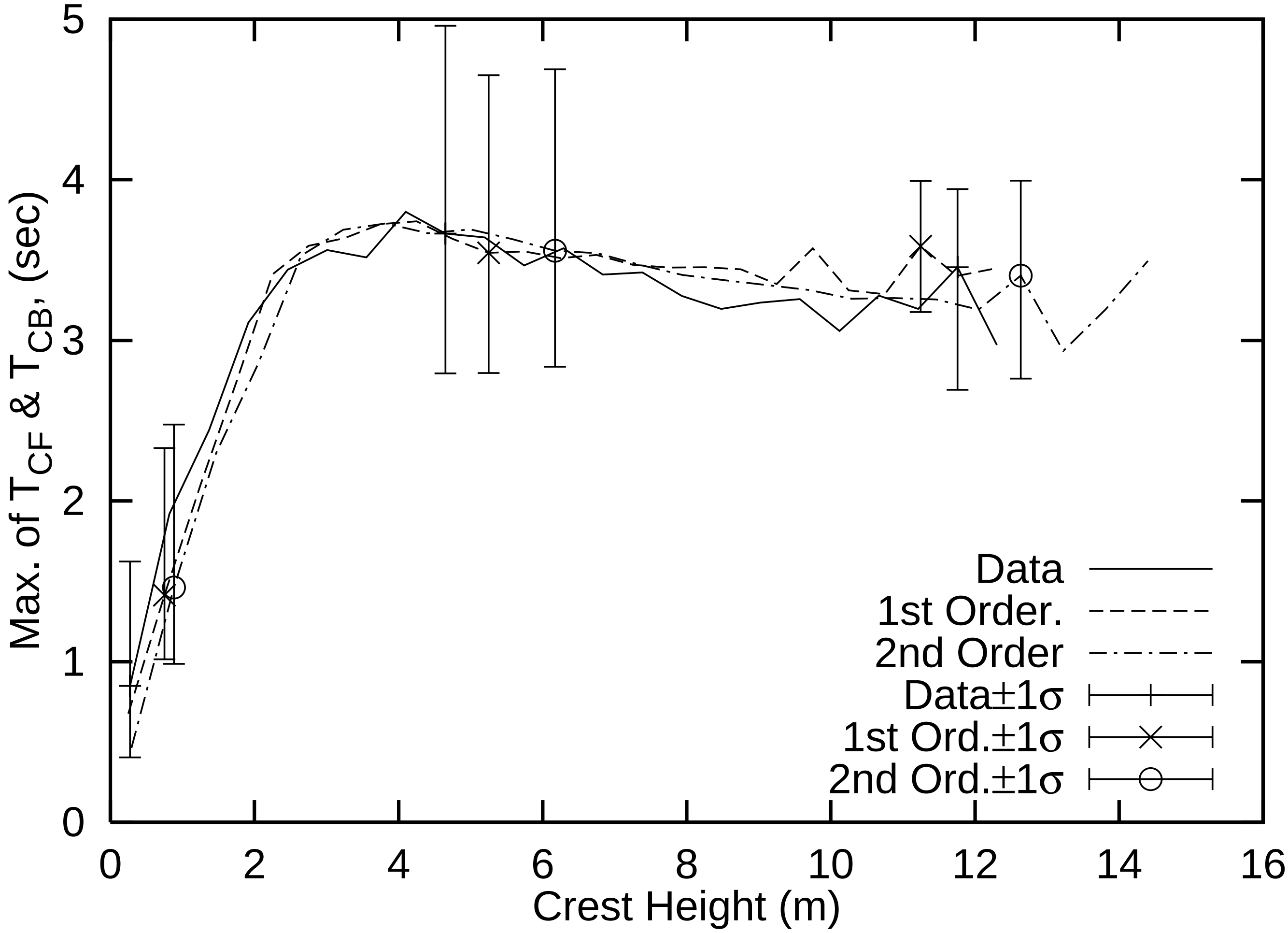
- Both wave crest and height underestimated marginally
- Conditional crest given wave height accurate nonetheless
- For accurate crest height model:
  - Use Stokes theory for mean crest (given height); estimate crest variability from data ("Noisy Stokes")
  - Use Hermite model with skewness from 2nd-order theory; estimate (enhanced) kurtosis from data ("Empirical Hermite")

Local Wave Statistics (contd)



Crest Period T_C vs. wave period T_W

Local Wave Statistics (contd)



Maximum of crest front and crest back periods vs. crest height

Conclusions

1. Convenient analytical expressions shown to match random 2nd-order waves

- wave skewness, kurtosis
- wave elevation/crest/height CDF

Hence: results for 2nd-order models available without simulation

2. Comparison between 2nd-order models and data:

- skewness: 2nd-order model ≈ field ≈ wavetank data
- kurtosis: 2nd-order model ≈ field < wavetank data
- elevation/crest/height: 2nd-order model ≈ field < wavetank data
- Profile shapes (C/H, steepness) accurately predicted

Recommendations: For accurate crest height model

- Use Stokes theory for mean crest (given height); estimate crest variability from data ("Noisy Stokes")
- Use Hermite model with skewness from 2nd-order theory; estimate (enhanced) kurtosis from data ("Empirical Hermite")