Smart Structures and Materials
and NDE for Health Monitoring
and Diagnostics An SPIE Event

The Modal Distribution Method

A New Statistical Algorithm For Analyzing Measured Acceleration Data

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Purpose

- Infer something about structure from measured acceleration data
- Separate individual modes
- Modal parameters are a proxy for structural behavior
- Modal frequencies are a function of structural properties
  - Mass : riser
  - Stiffness : Structural Health Monitoring

Underlying Assumptions

- **Independent individual modes** of vibration
- **T-test** applicable to each modal comparison
- **The number of cycles** is proportional to frequency and different for each mode

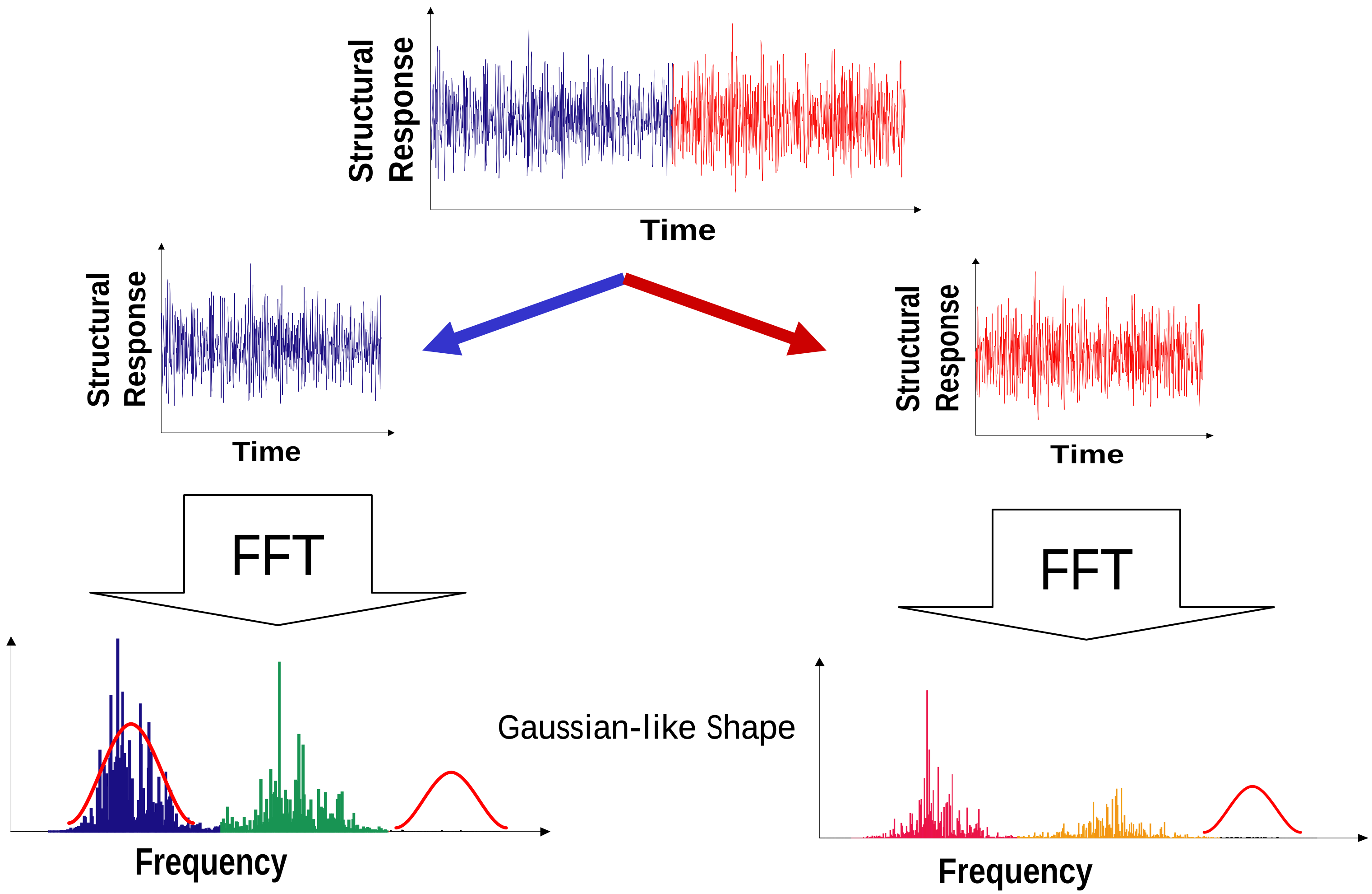
Advantage of Modal Distribution Method

- Applicable Any Number of Modes
- Vibration Response
- Independent Modes
- Roughly Guessed Initial Modal Frequencies
- Numerical Significance Level

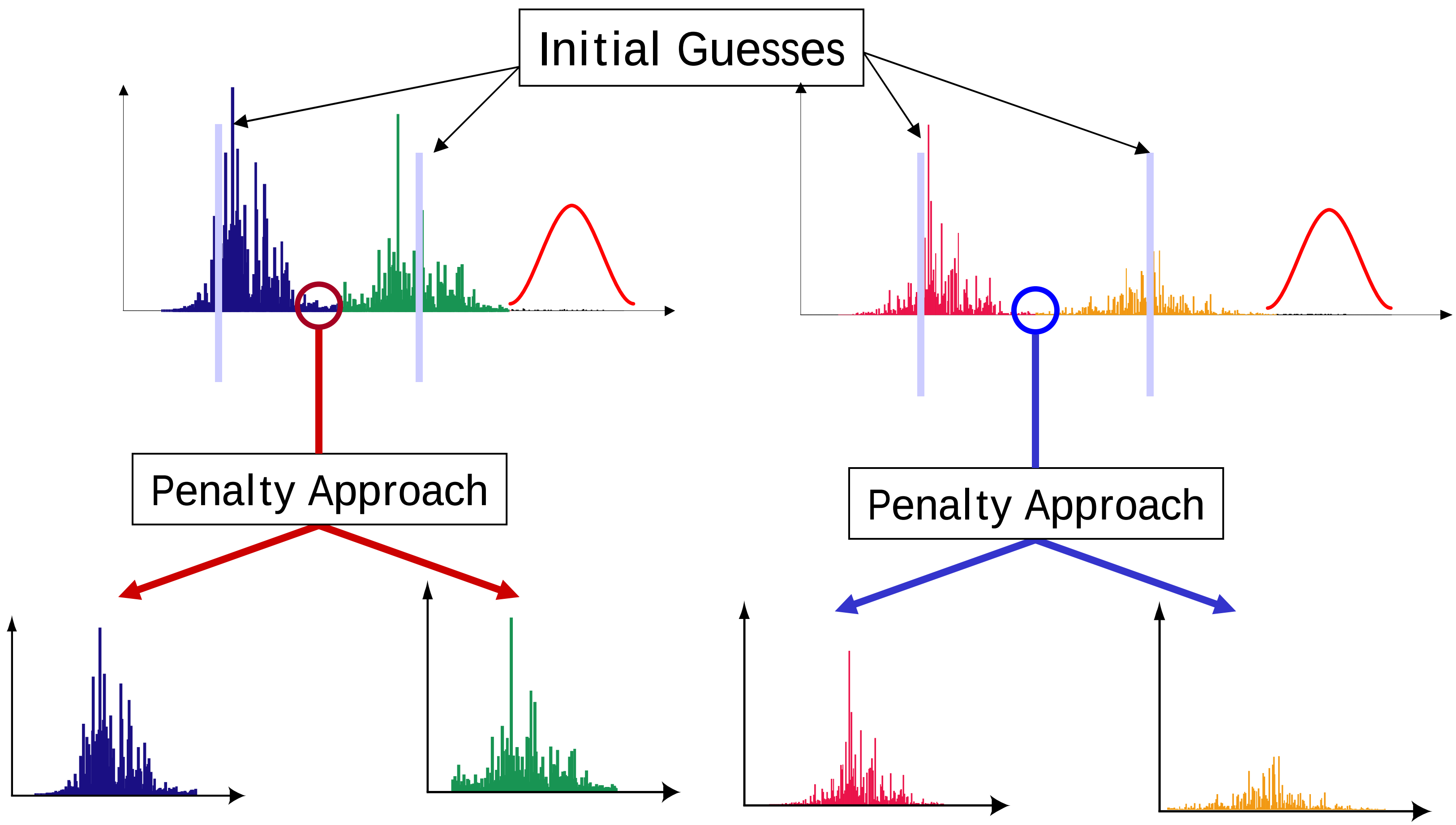
OVERVIEW OF NEW METHOD

1. Power Spectra of Measured Time-history
2. Separation of Modal Distributions
3. Isolated Modal Distribution and Probability Density Function
4. Statistical Comparison between Two Power Spectra

Measured Time History and Power Spectra



Separation of Modal Distributions

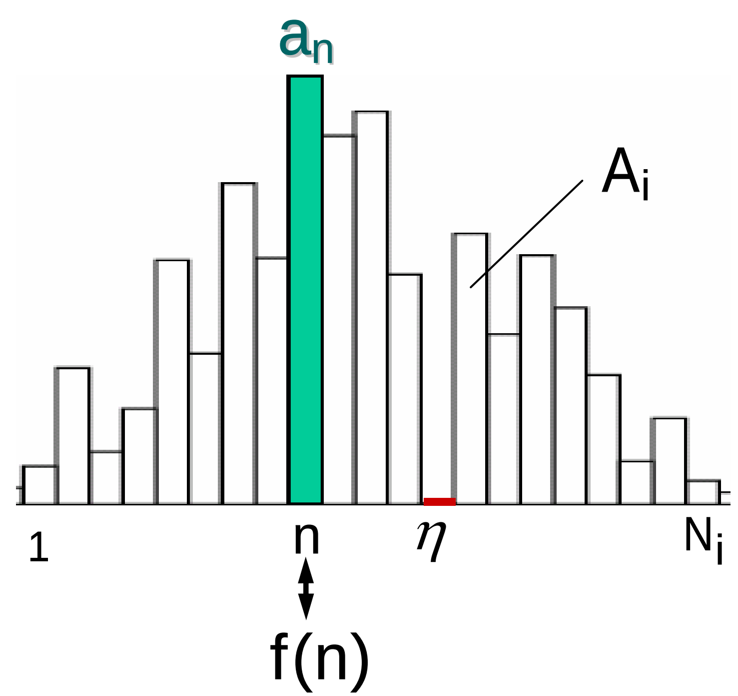


Modal Distribution as Probability Density Function

- i'th mode

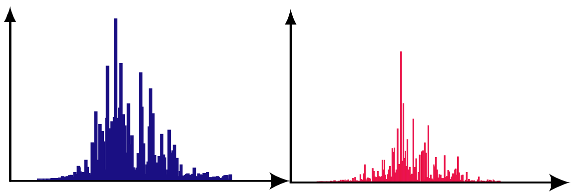
$$P_i(e_n) = \frac{a_n}{A_i}$$

$$A_i = \sum_{k=1}^{N_i} a_k$$

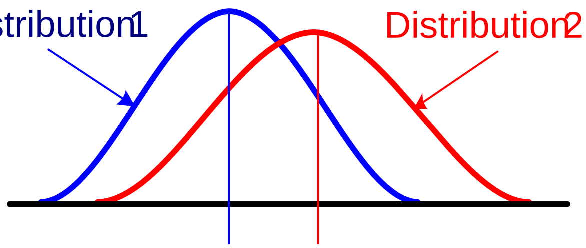


Statistical Comparison between Two Distributions

- Comparing Same Modes in Different Windows

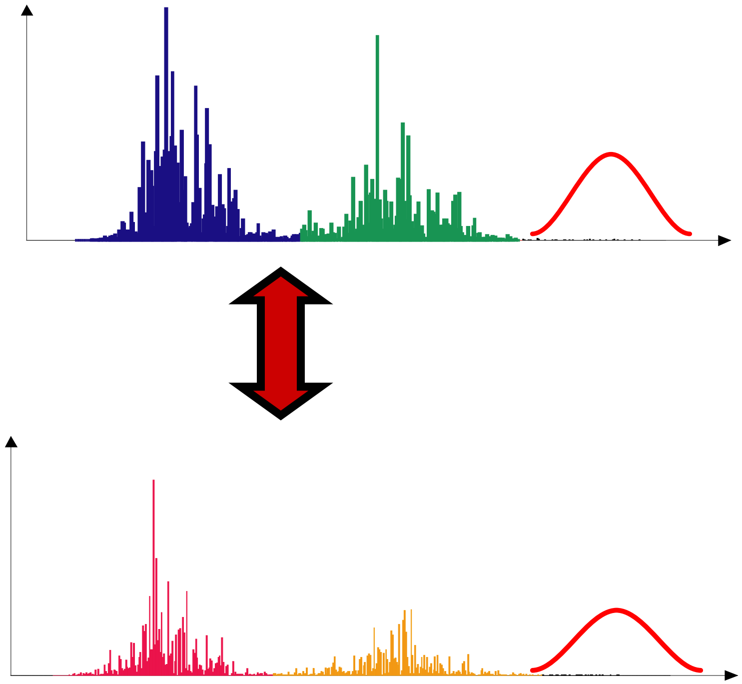


- Comparing Distributions
  - Resulting P-Value
  - Difference by random chance



Statistical Comparison between Two Spectra

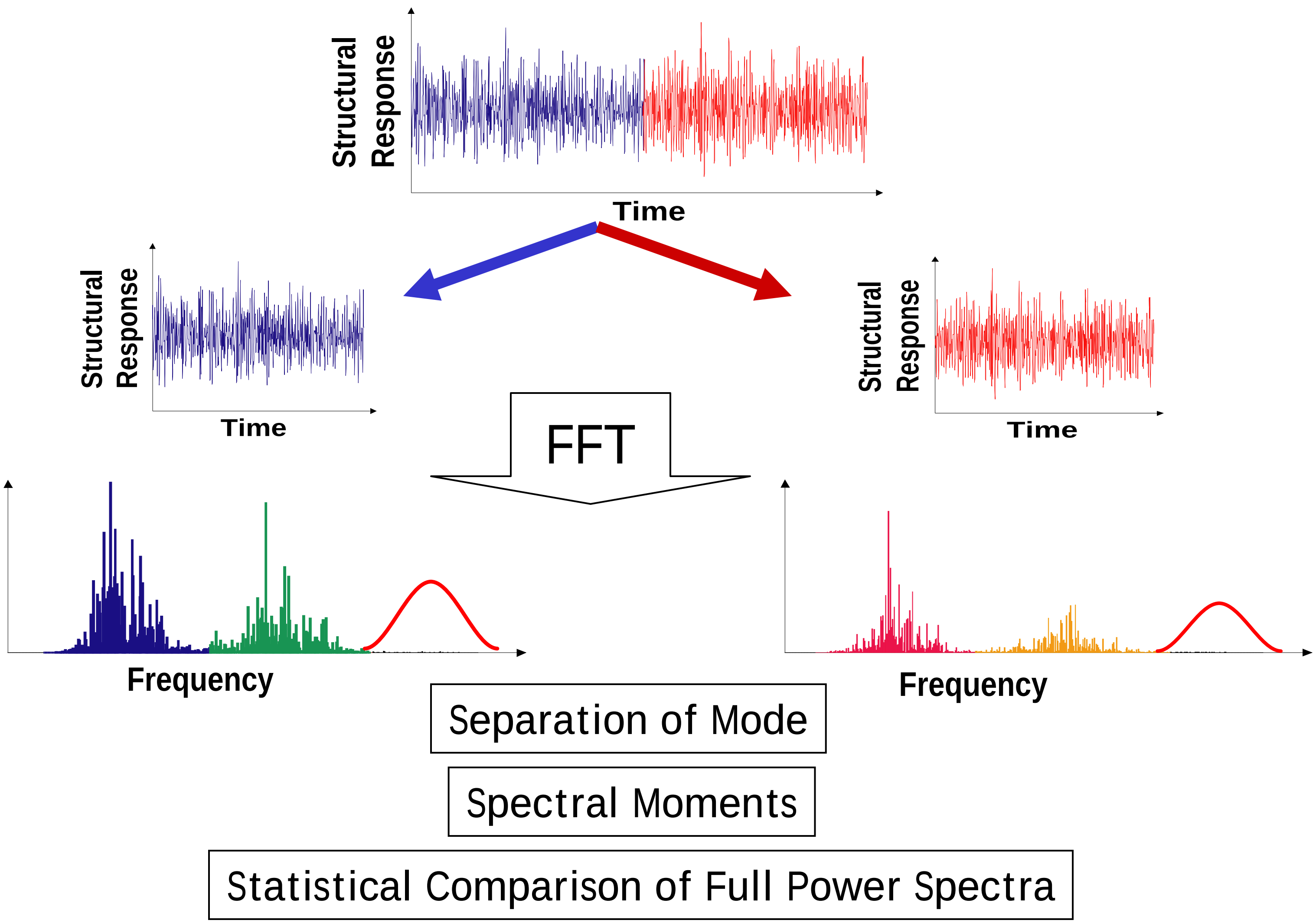
- Overall Comparison



Power Spectrum of 1st Window

Power Spectrum of 2nd Window

Summary of Modal Distribution Method

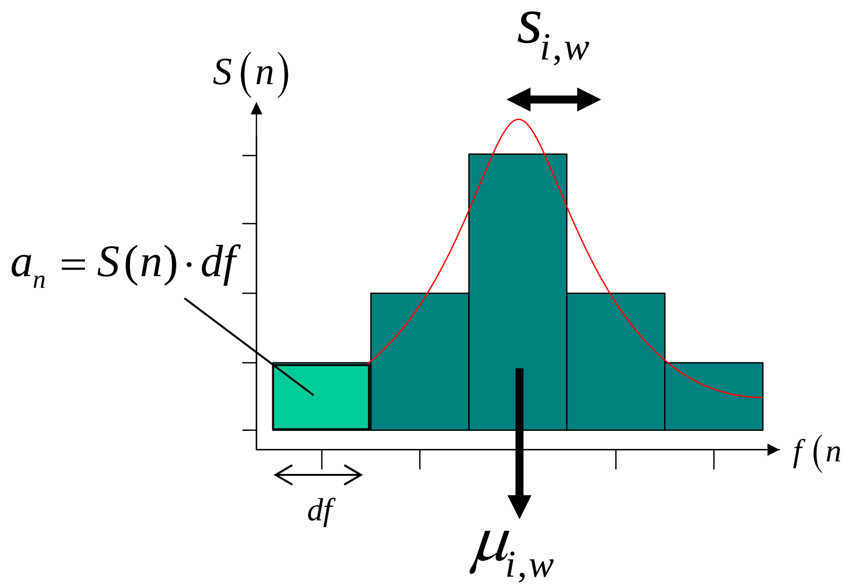


COMPUTATIONAL DETAILS

1. Spectral Moments Calculation
2. Statistical Comparison Between Two Modal Distributions
3. Overall Comparison Between Two Spectra

Calculation of Spectral Moments

- Mean and Variance
  -Direct Calculation
  Using Geometry



$$\mu_{i,w} = \frac{1}{A_i} \sum_{n=1}^{N_i} a_n f(n)$$

$$s_{i,w}^2 = \frac{1}{A_i} \sum_{n=1}^{N_i} a_n \left[f(n) - \mu_{i,w} \right]^2$$

Statistical Comparison Between Two Distributions

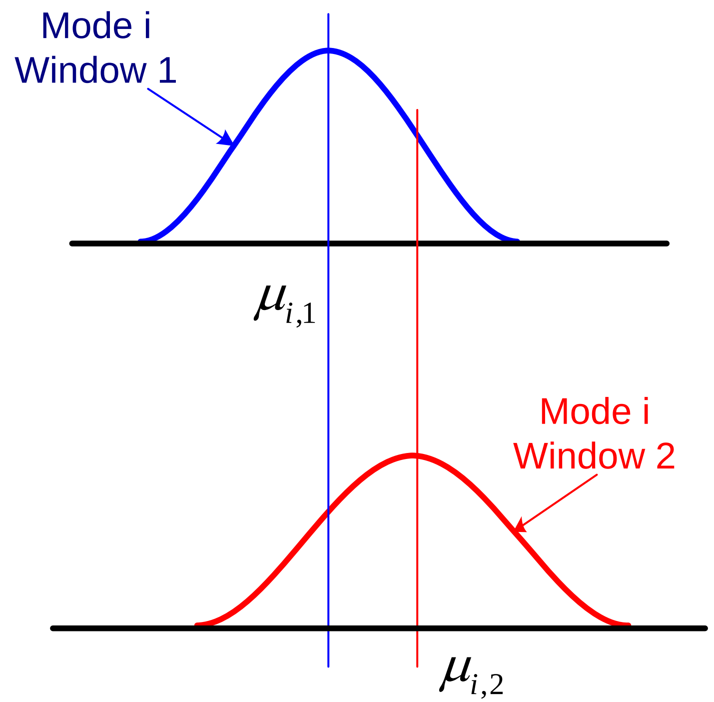
- Modal comparison

$$T_i = \frac{\mu_{i,1} - \mu_{i,2}}{s_i}$$

where
$$N_{i,w} = T_w \cdot \mu_{i,w}$$

$$s_i^2 = \frac{s_{i,1}^2}{N_{i,1}} + \frac{s_{i,2}^2}{N_{i,2}}$$

$$DOF_i = \frac{s_i^4}{\frac{\left(s_{i,1}^2 / N_{i,1}\right)^2}{N_{i,1} - 1} + \frac{\left(s_{i,2}^2 / N_{i,2}\right)^2}{N_{i,2} - 1}}$$



Overall Comparison of Response

$$T = \frac{\Delta\mu}{s}$$

- Multi Modes

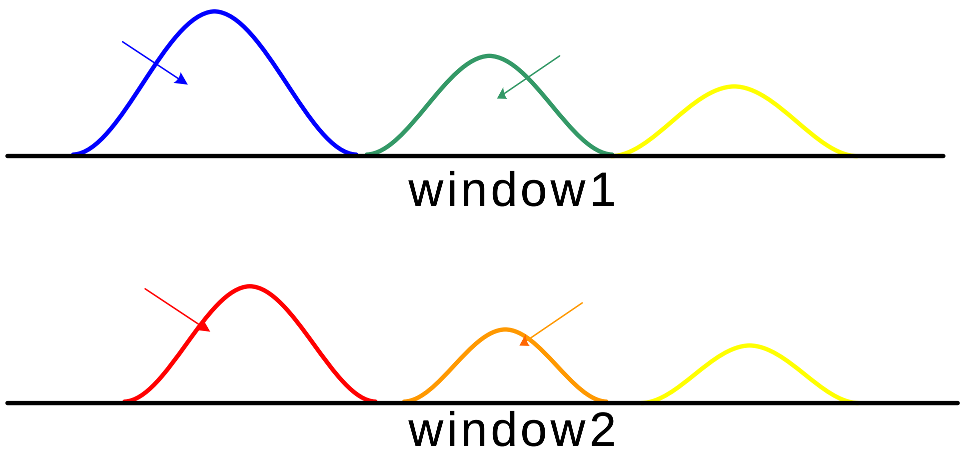
$$\Delta\mu = 0.5\left(E_{1,1} + E_{1,2}\right)\Delta\mu_1 + 0.5\left(E_{2,1} + E_{2,2}\right)\Delta\mu_2 + \cdots$$

$$s^2 = 0.5\left(E_{1,1} + E_{1,2}\right)s_1^2 + 0.5\left(E_{2,1} + E_{2,2}\right)s_2^2 + \cdots$$

$$DOF = DOF_1 + DOF_2 + \cdots$$

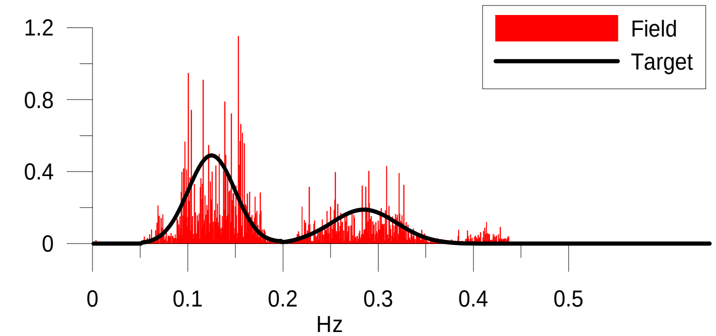
$$E_{i,w} = \frac{A_{i,w}}{A_{1,w} + A_{2,w} + \cdots}$$

$$\Delta\mu_i = \left|\mu_{i,1} - \mu_{i,2}\right|$$



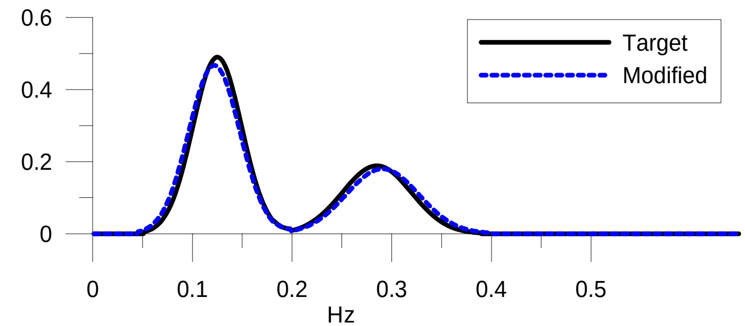
EXAMPLE: SIMULATED TIME-HISTORY

- Compute Spectrum of Acceleration Time-history Measured in Real-field Riser

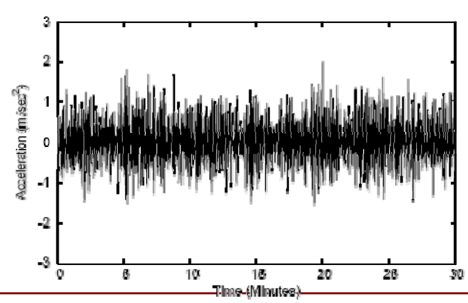


- Idealize Target Spectrum

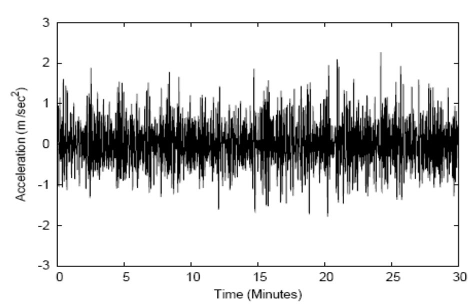
- Modify Spectra



- Simulate Time-history

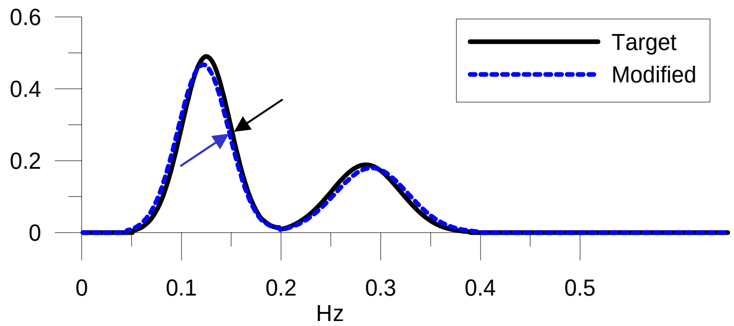


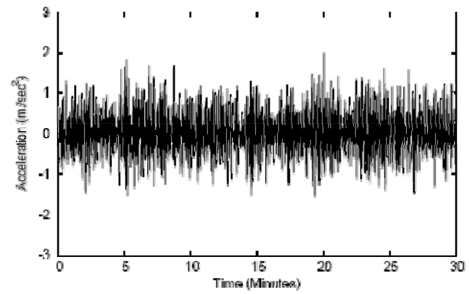
Original Target Spectrum



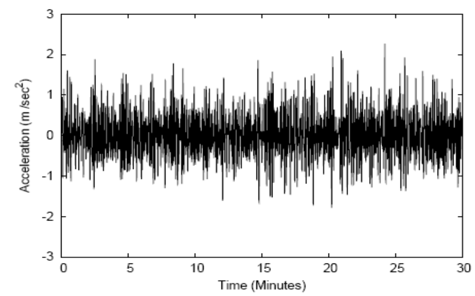
Modified Target Spectrum

RESULTS: Combined Change

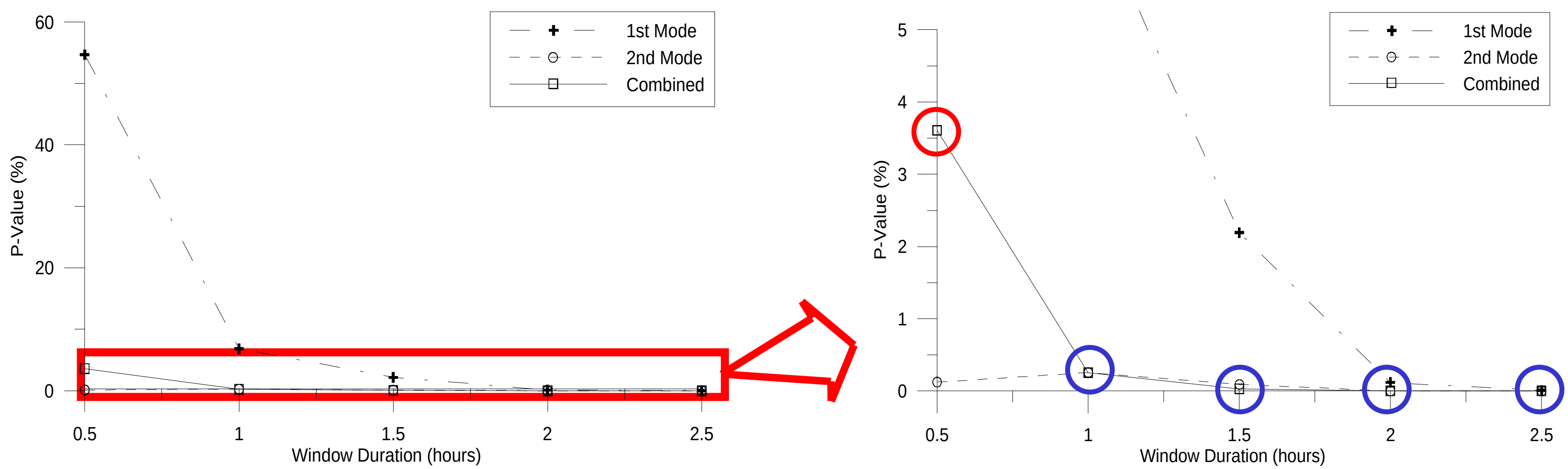




Original Target Spectrum



Modified Target Spectrum



CONCLUSIONS

- A newly proposed **Modal Distribution Method (MDM)** has been presented.
- The MDM is **general** and may find a broad variety of applications, particularly Structural Health Monitoring.
- The MDM is **effective** at detecting subtle changes in mean modal frequencies with same mean and variance of underlying processes
- Only a **30 minute data window** is required to detect **the subtle changes shown** with **3.6% confidence level**.
- Using more than one hour does not significantly improve P-value.

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Any Comments

and/or

Questions

A&M Galveston